

# EE 330

## Lecture 37

### Digital Circuit Design

- Characterization of CMOS Inverter
- One device sizing strategy

# Exam Schedule

Exam 1	Friday Sept 24
Exam 2	Friday Oct 22
Exam 3	Friday Nov 19
Final	Tues Dec 14 12:00 p.m.

Photo courtesy of the director of the National Institute of Health ( NIH)



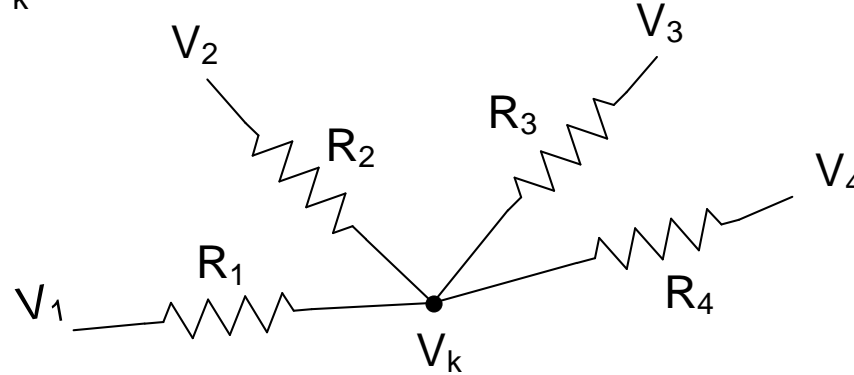
As a courtesy to fellow classmates, TAs, and the instructor

**Wearing of masks during lectures and in the laboratories for this course would be appreciated irrespective of vaccination status**

# Nodal Analysis (A Brief Review)

Widely used to analyze electronic circuits – and for good reason!

Example: Determine  $V_k$



From KCL

$$\left(\frac{V_k - V_1}{R_1}\right) + \left(\frac{V_k - V_2}{R_2}\right) + \left(\frac{V_k - V_3}{R_3}\right) + \left(\frac{V_k - V_4}{R_4}\right) = 0$$

$$V_k \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}\right) = \frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3} + \frac{V_4}{R_4}$$

$$V_k = V_1 \frac{1}{R_1 \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}\right)} + V_2 \frac{1}{R_2 \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}\right)} + V_3 \frac{1}{R_3 \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}\right)} + V_4 \frac{1}{R_4 \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}\right)}$$

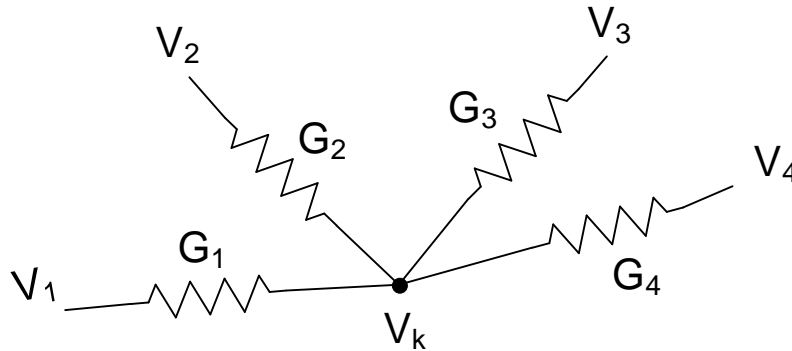
$$V_k = V_1 \frac{R_2 R_3 R_4}{(R_2 R_3 R_4 + R_1 R_3 R_4 + R_2 R_1 R_4 + R_2 R_3 R_1)} + V_2 \frac{R_1 R_3 R_4}{(R_2 R_3 R_4 + R_1 R_3 R_4 + R_2 R_1 R_4 + R_2 R_3 R_1)} + V_3 \frac{R_2 R_1 R_4}{(R_2 R_3 R_4 + R_1 R_3 R_4 + R_2 R_1 R_4 + R_2 R_3 R_1)} + V_4 \frac{R_2 R_3 R_1}{(R_2 R_3 R_4 + R_1 R_3 R_4 + R_2 R_1 R_4 + R_2 R_3 R_1)}$$

Time consuming and tedious for even simple circuits

# Nodal Analysis (A Brief Review)

Widely used to analyze electronic circuits – and for good reason!

Example: Determine  $V_k$



From KCL 
$$V_k (G_1 + G_2 + G_3 + G_4) = G_1 V_1 + G_2 V_2 + G_3 V_3 + G_4 V_4$$

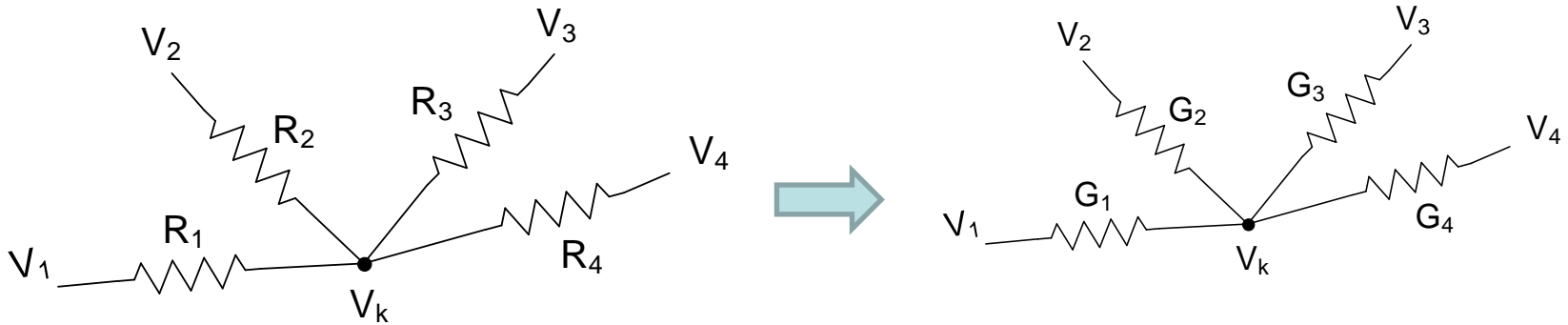
$$V_k = V_1 \frac{G_1}{G_1 + G_2 + G_3 + G_4} + V_2 \frac{G_2}{G_1 + G_2 + G_3 + G_4} + V_3 \frac{G_3}{G_1 + G_2 + G_3 + G_4} + V_4 \frac{G_4}{G_1 + G_2 + G_3 + G_4}$$

Often much simpler to work with conductances than with resistances!

And expressions much simpler

# Nodal Analysis (A Brief Review)

Widely used to analyze electronic circuits – and for good reason!



And expressions much simpler (compare in standard rational fraction form)

$$V_k = V_1 \frac{R_2 R_3 R_4}{(R_2 R_3 R_4 + R_1 R_3 R_4 + R_2 R_1 R_4 + R_2 R_3 R_1)} + V_2 \frac{R_1 R_3 R_4}{(R_2 R_3 R_4 + R_1 R_3 R_4 + R_2 R_1 R_4 + R_2 R_3 R_1)} + V_3 \frac{R_2 R_1 R_4}{(R_2 R_3 R_4 + R_1 R_3 R_4 + R_2 R_1 R_4 + R_2 R_3 R_1)} + V_4 \frac{R_2 R_3 R_1}{(R_2 R_3 R_4 + R_1 R_3 R_4 + R_2 R_1 R_4 + R_2 R_3 R_1)}$$

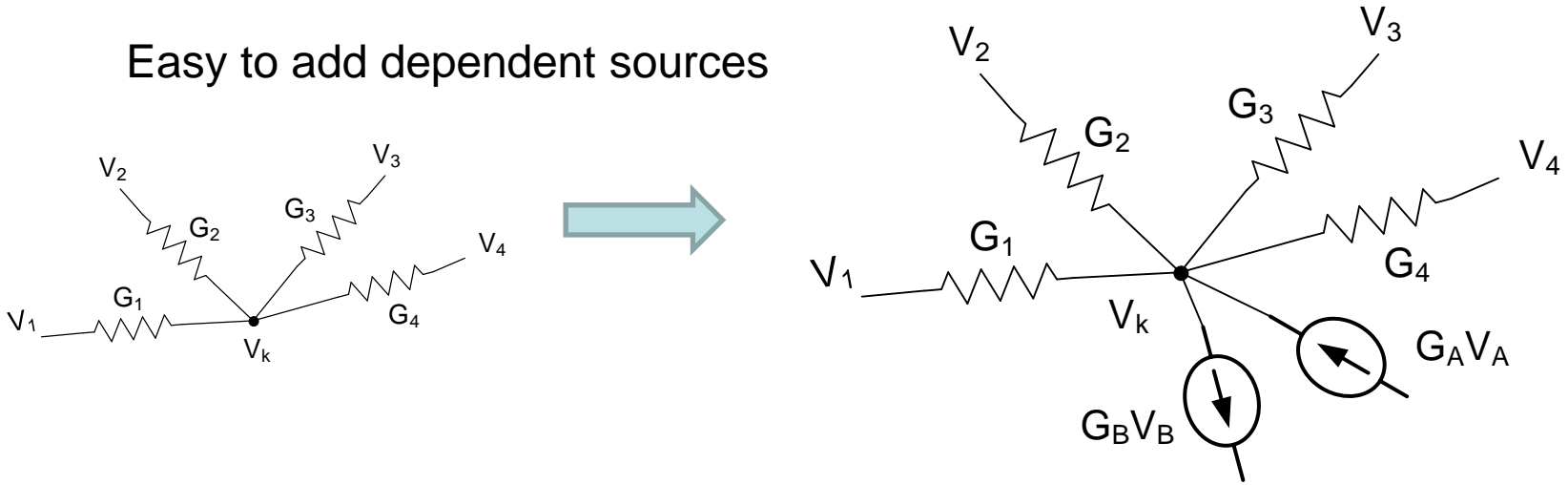
$$V_k = V_1 \frac{G_1}{G_1 + G_2 + G_3 + G_4} + V_2 \frac{G_2}{G_1 + G_2 + G_3 + G_4} + V_3 \frac{G_3}{G_1 + G_2 + G_3 + G_4} + V_4 \frac{G_4}{G_1 + G_2 + G_3 + G_4}$$

# Nodal Analysis (A Brief Review)

Widely used to analyze electronic circuits – and for good reason!

Example: Determine  $V_k$

Easy to add dependent sources



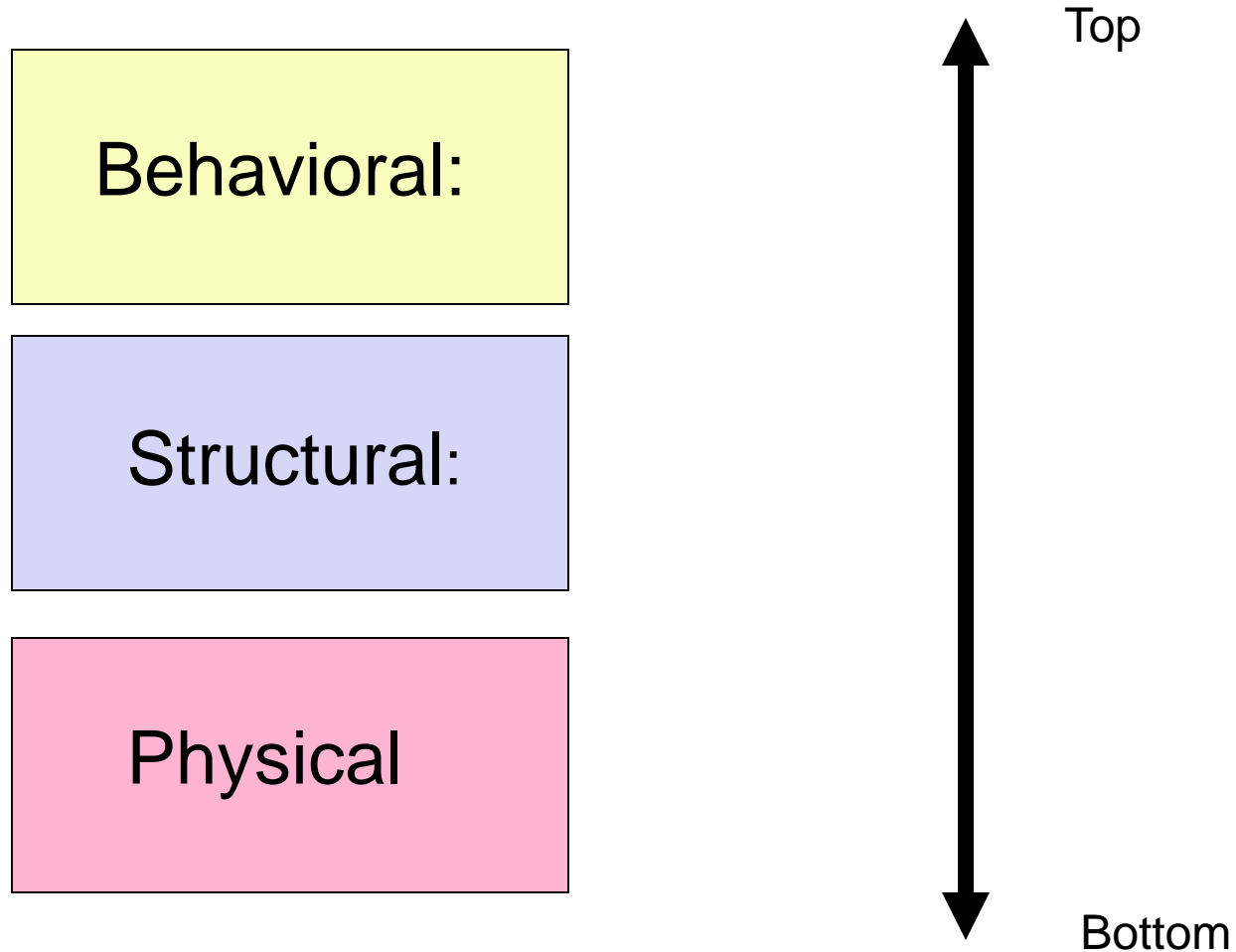
$$\text{From KCL} \quad V_k (G_1 + G_2 + G_3 + G_4) + G_B V_B - G_A V_A = G_1 V_1 + G_2 V_2 + G_3 V_3 + G_4 V_4$$

$$V_k = V_1 \frac{G_1}{G_1 + G_2 + G_3 + G_4} + V_2 \frac{G_2}{G_1 + G_2 + G_3 + G_4} + V_3 \frac{G_3}{G_1 + G_2 + G_3 + G_4} + V_4 \frac{G_4}{G_1 + G_2 + G_3 + G_4} + V_A \frac{G_A}{G_1 + G_2 + G_3 + G_4} - V_B \frac{G_B}{G_1 + G_2 + G_3 + G_4}$$

Often much simpler to work with conductances than with resistances!

Do we really need the concept of both a resistor and a conductor?





# Hierarchical Digital Design Domains:



Multiple Levels of Abstraction



# Digital Circuit Design

-  • Hierarchical Design
  -  • Basic Logic Gates
  -  • Properties of Logic Families
  -  • Characterization of CMOS Inverter
    - Static CMOS Logic Gates
      - Ratio Logic
    - Propagation Delay
      - Simple analytical models
        - FI/OD
        - Logical Effort
      - Elmore Delay
    - Sizing of Gates
      - The Reference Inverter
  - Propagation Delay with Multiple Levels of Logic
  - Optimal driving of Large Capacitive Loads
  - Power Dissipation in Logic Circuits
  - Other Logic Styles
  - Array Logic
  - Ring Oscillators
- 

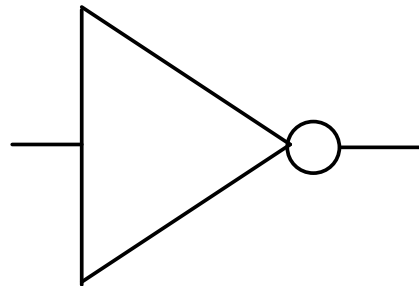
 done

 partial

Review from last lecture

# The basic logic gates

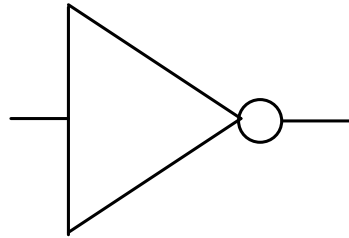
It suffices to characterize the inverter of a logic family and then express the performance of other gates in that family in terms of the performance of the inverter.



What characteristics are required and desirable for an inverter to form the basis for a useful logic family?

Review from last lecture

What are the logic levels for a given inverter of for a given logic family?

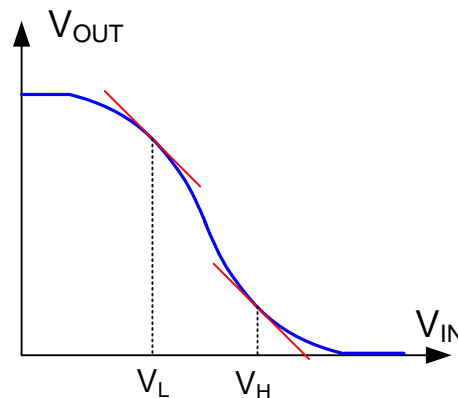


$V_H=?$

$V_L=?$

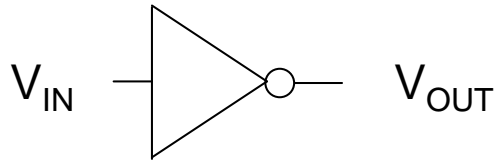
Can we legislate them ?

- Some authors choose to define them based upon specific features of inverter
- Analytical expressions may be complicated
- But what if the circuit does not interpret them the same way they are defined !!



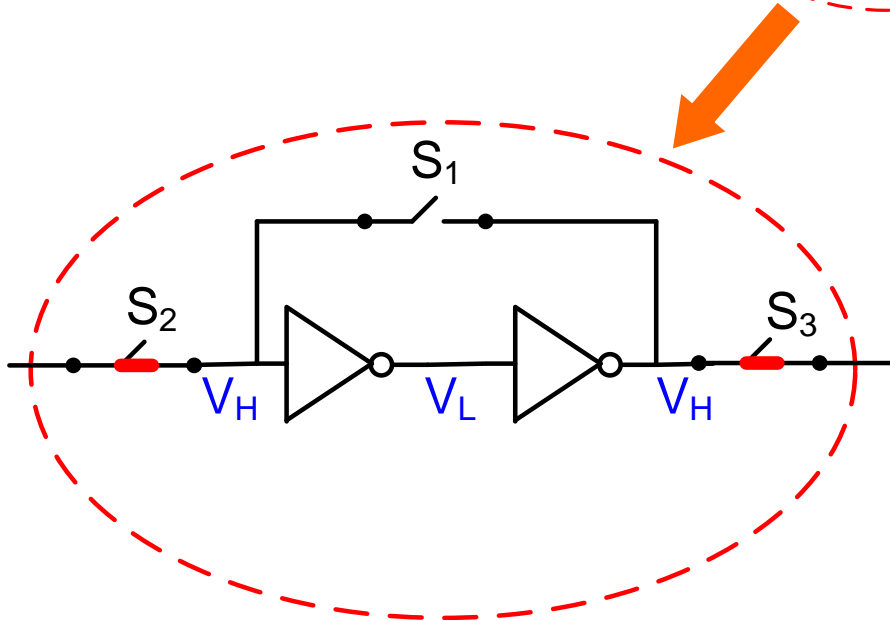
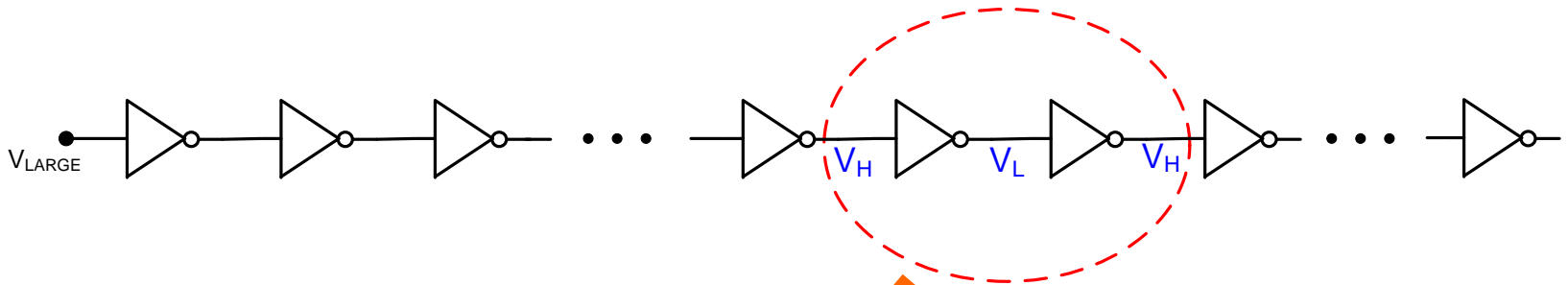
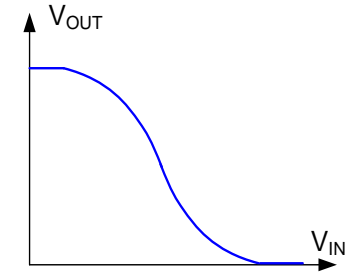
# Review from last lecture

## Ask the inverter how it will interpret logic levels

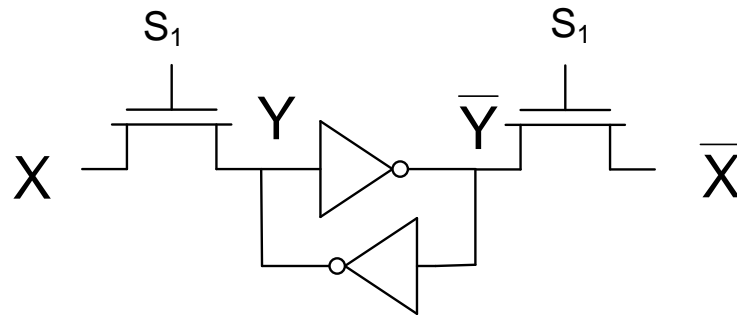
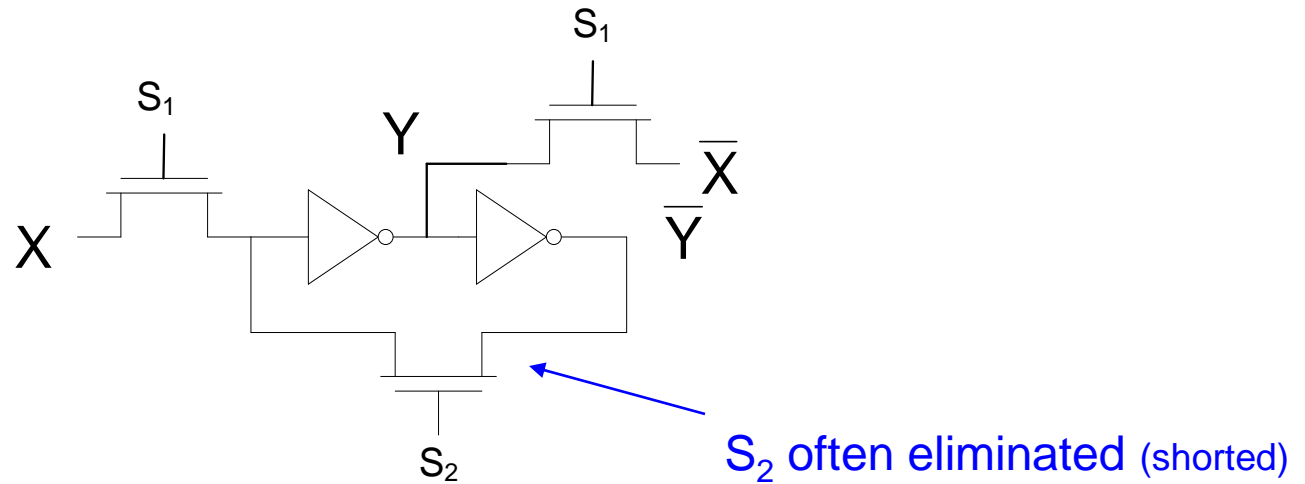


$V_H=?$

$V_L=?$



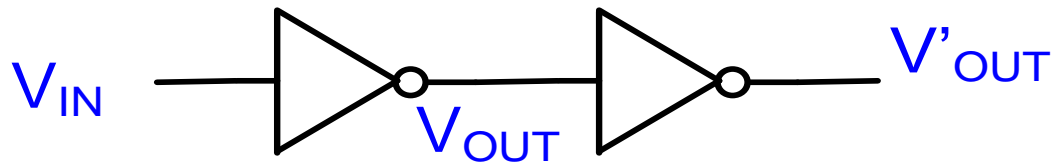
# The two-inverter loop



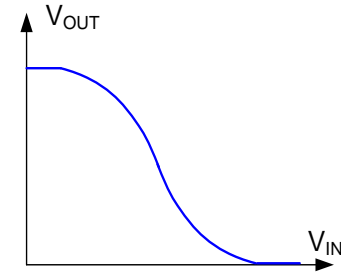
Standard 6-transistor SRAM Cell

## Review from last lecture

Ask the inverter how it will interpret logic levels



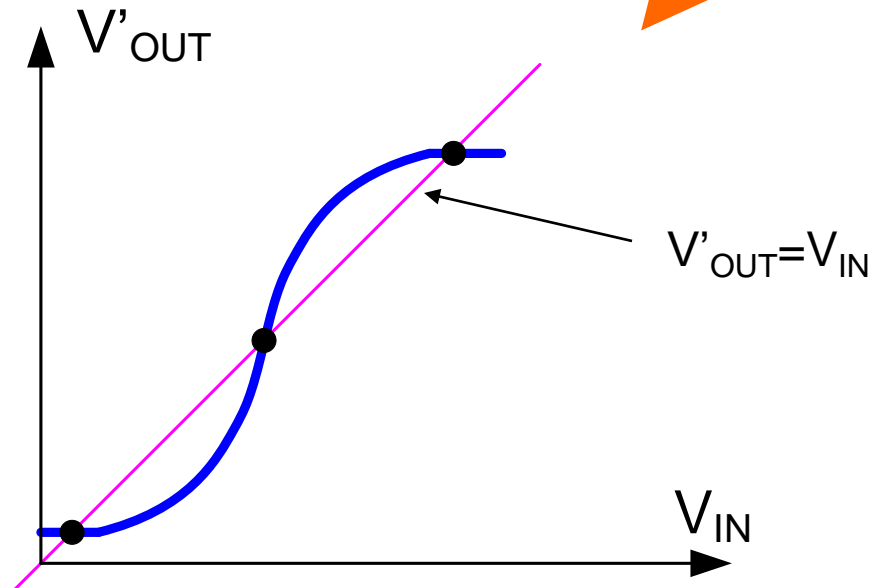
Inverter pair



$V_H=?$

$V_L=?$

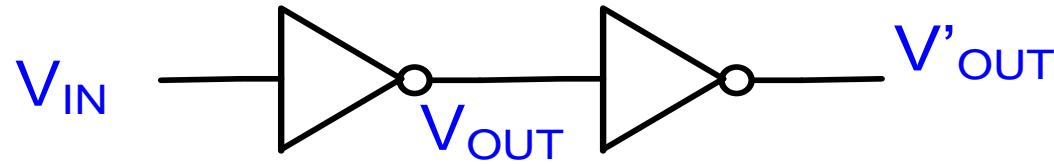
$V_H$  and  $V_L$  will be on the intersection of the transfer characteristics of the inverter pair (IPTC) and the  $V'_{OUT}=V_{IN}$  line



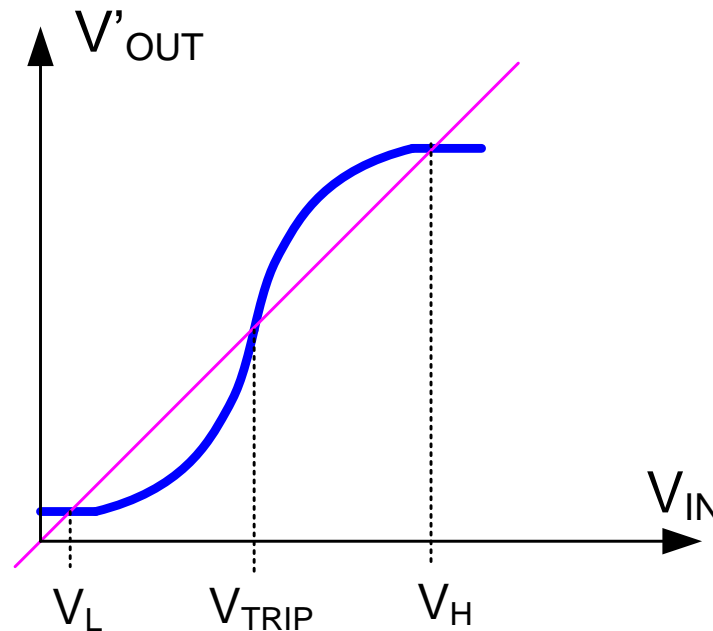
$V_H$  and  $V_L$  often termed the "1" and "0" states

Review from last lecture

Ask the inverter how it will interpret logic levels



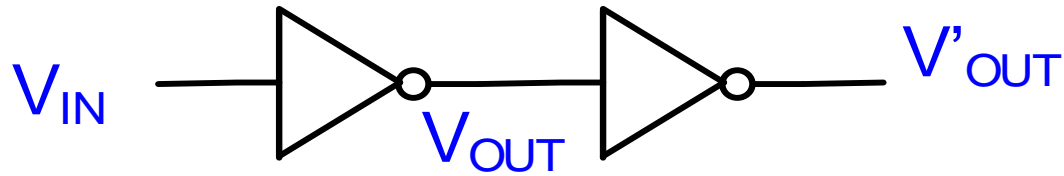
Inverter pair



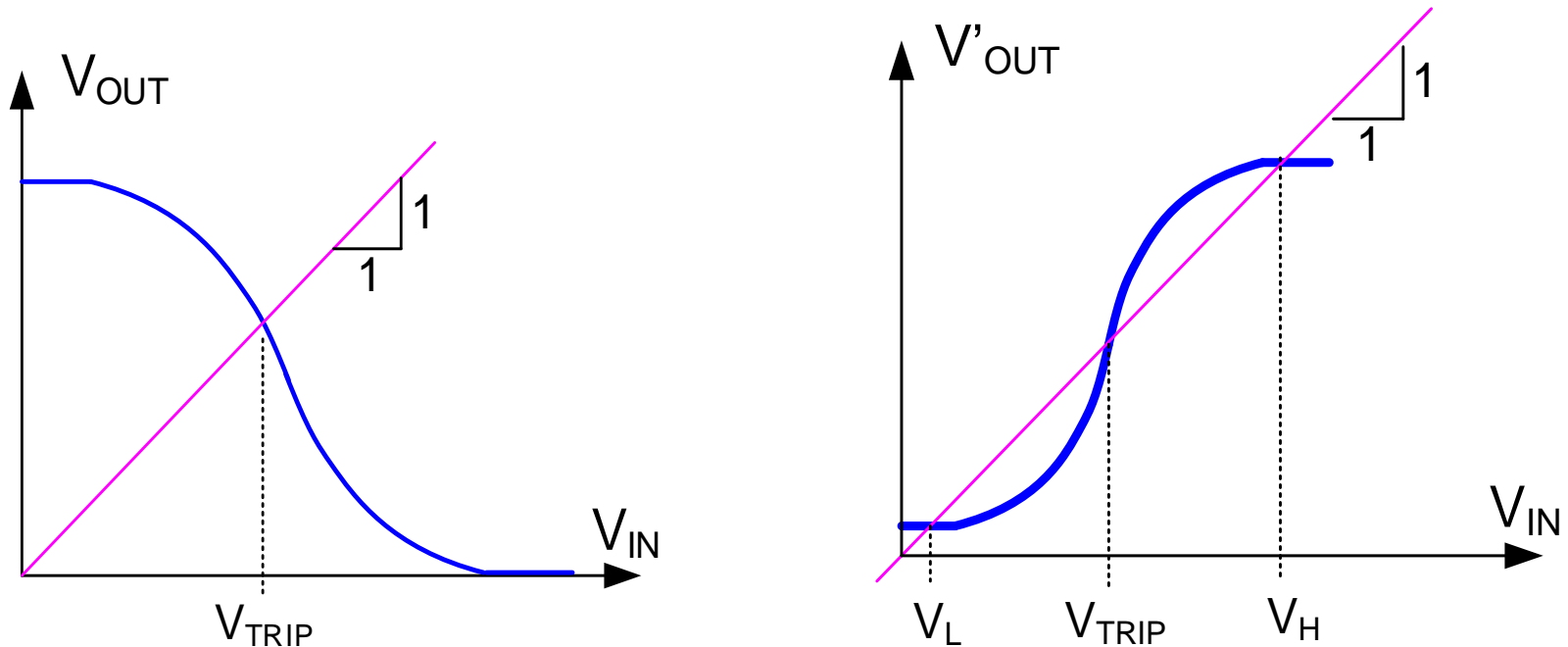
When  $V'_{OUT} = V_{IN}$ ,  $V_H$  and  $V_L$  are stable operating points,  $V_{TRIP}$  is a quasi-stable operating point

Observe: slope of IPTC is greater than 1 at  $V_{TRIP}$  and less than 1 at  $V_H$  and  $V_L$

## Observation



When  $V_{OUT}=V_{IN}$  for the inverter,  $V'_{OUT}$  is also equal to  $V_{IN}$ . Thus the intersection point for  $V_{OUT}=V_{IN}$  in the inverter transfer characteristics (ITC) is also an intersection point for  $V'_{OUT}=V_{IN}$  in the inverter-pair transfer characteristics (IPTC)



**Implication:** Inverter characteristics can be used directly to obtain  $V_{TRIP}$



# Logic Family Characteristics

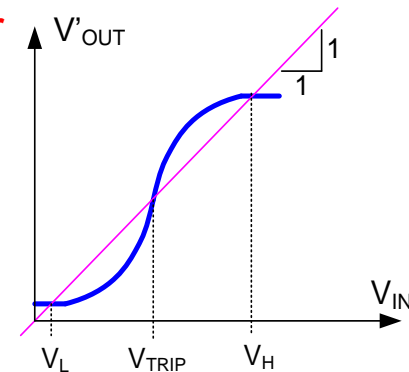
What properties of an inverter are necessary for it to be useful for building a two-level logic family?

The inverter-pair transfer characteristics must have three unique intersection points with the  $V'_{OUT} = V_{IN}$  line

What are the logic levels for a given inverter or for a given logic family?

The two extreme intersection points of the inverter-pair transfer characteristics with the  $V'_{OUT} = V_{IN}$  line

Can we legislate  $V_H$  and  $V_L$  for a logic family? **No!**

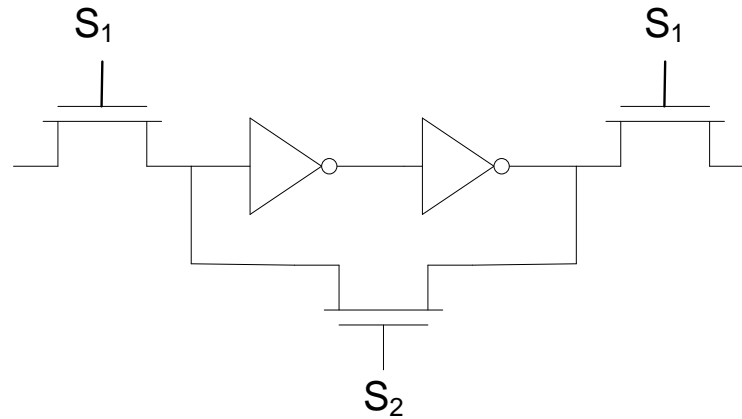


What other properties of the inverter are desirable?

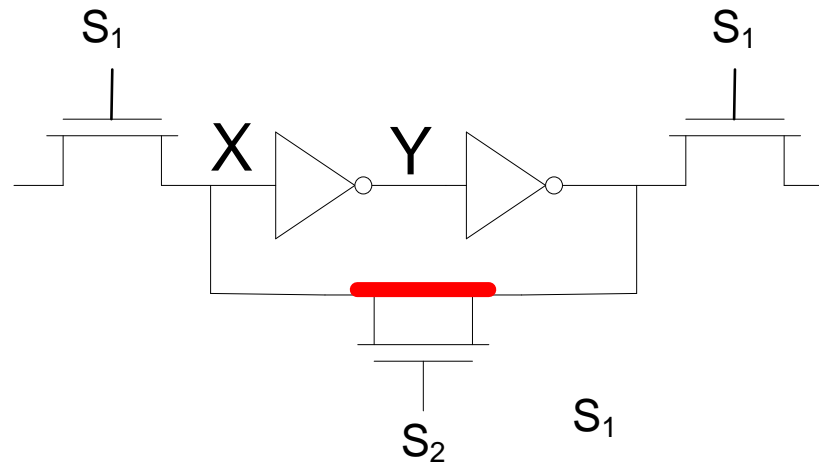
Reasonable separation between  $V_H$  and  $V_L$  (enough separation so that noise does not cause circuit to interpret level incorrectly)

$$V_{TRIP} \approx \frac{V_H + V_L}{2} \quad (\text{to provide adequate noise immunity and process insensitivity})$$

# What happens near the quasi-stable operating point?

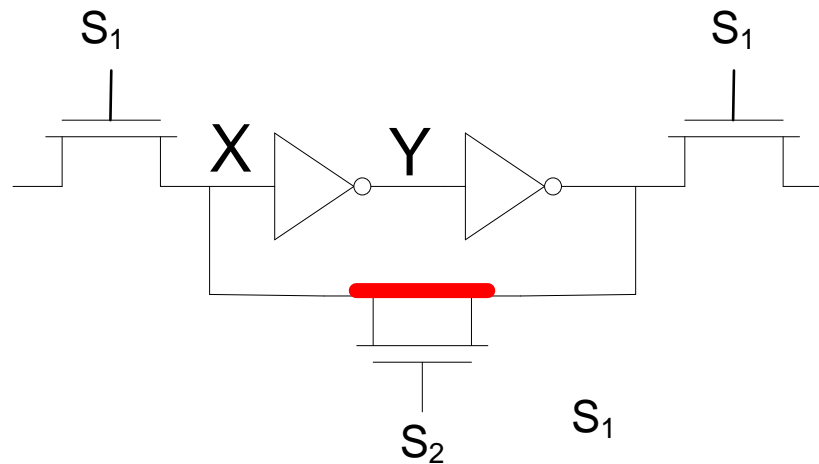


$S_2$  closed and  $X=Y=V_{TRIP}$



# What happens near the quasi-stable operating point?

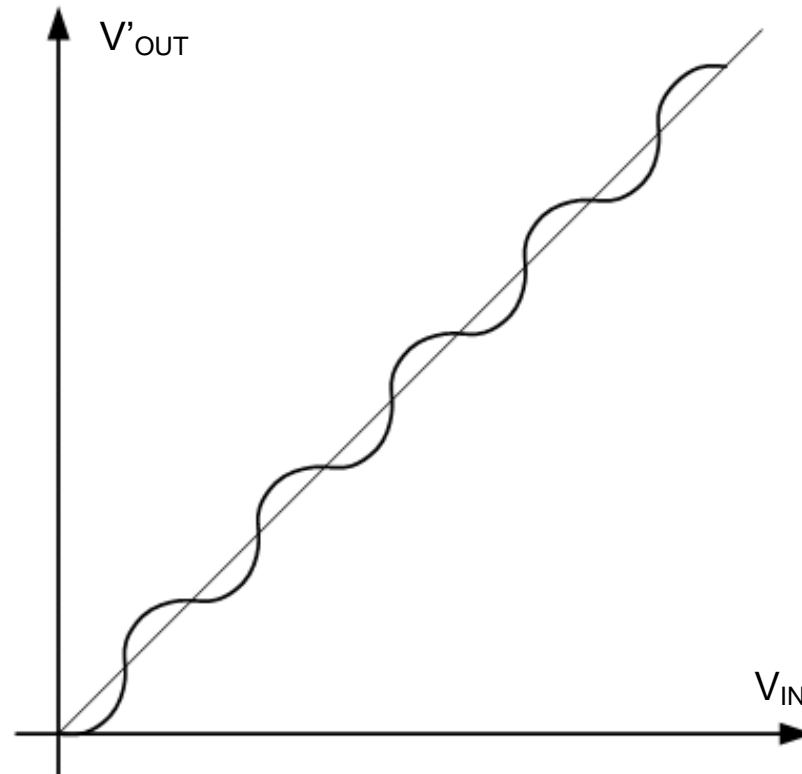
$S_2$  closed and  $X=Y=V_{TRIP}$



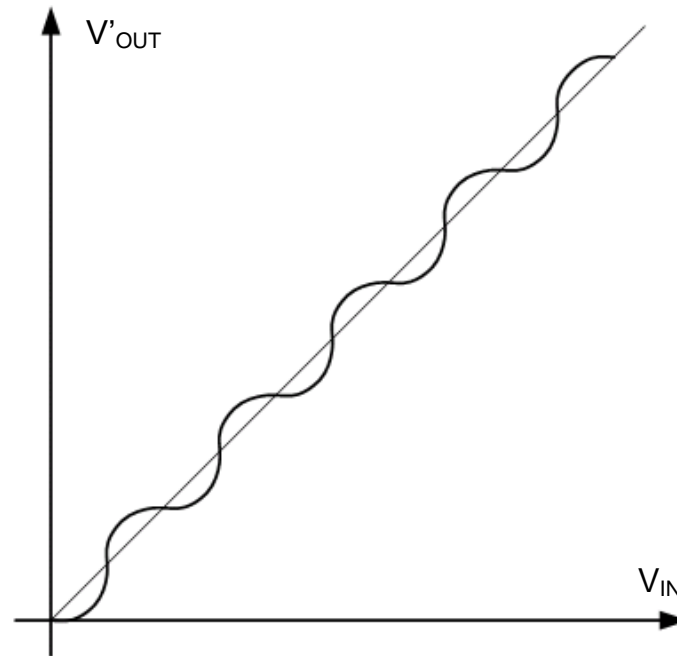
If  $X$  decreases even very slightly, will move to the  $X=0, Y=1$  state (very fast)

If  $X$  increases even very slightly, will move to the  $X=1, Y=0$  state (very fast)

What if the inverter pair had the following transfer characteristics?



# What if the inverter pair had the following transfer characteristics?

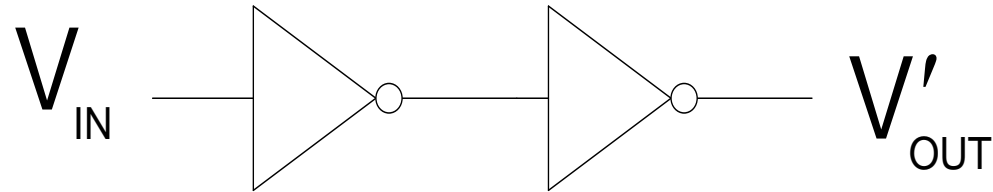


Multiple levels of logic

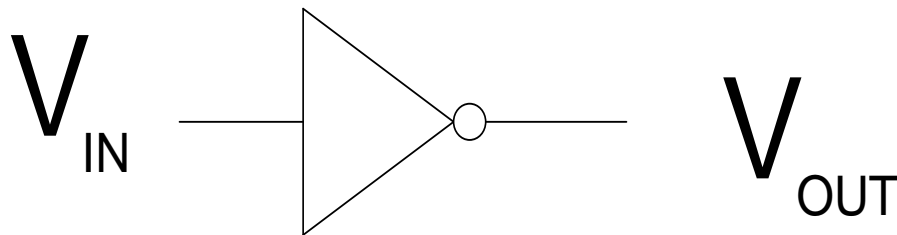
Every intersection point with slope  $< 1$  is a stable point

Every intersection point with slope  $> 1$  is a quasi-stable point

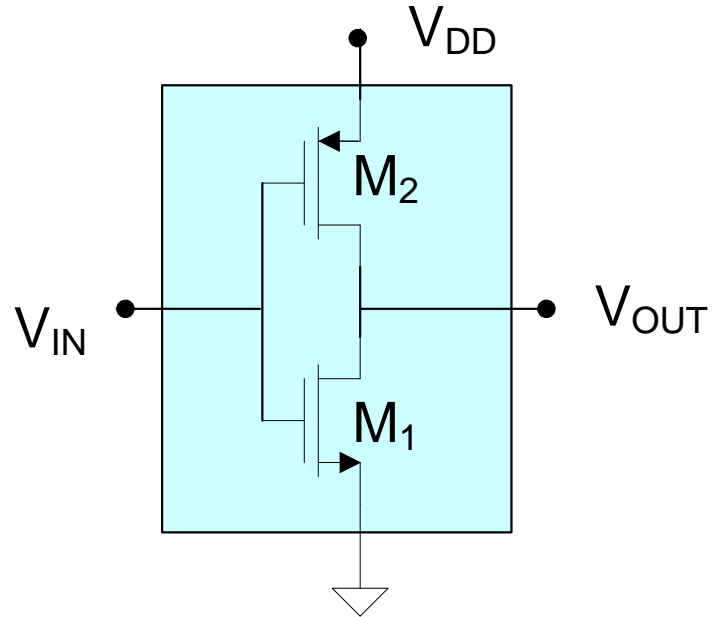
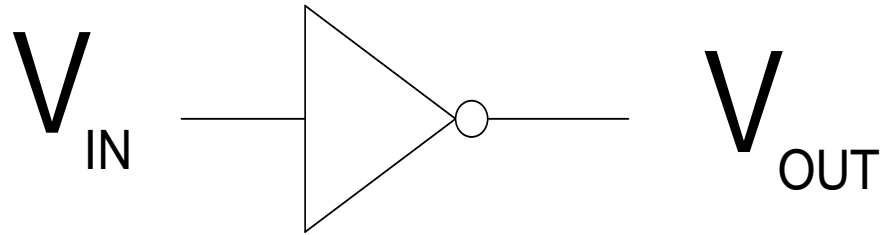
# What are the transfer characteristics of the static CMOS inverter pair?

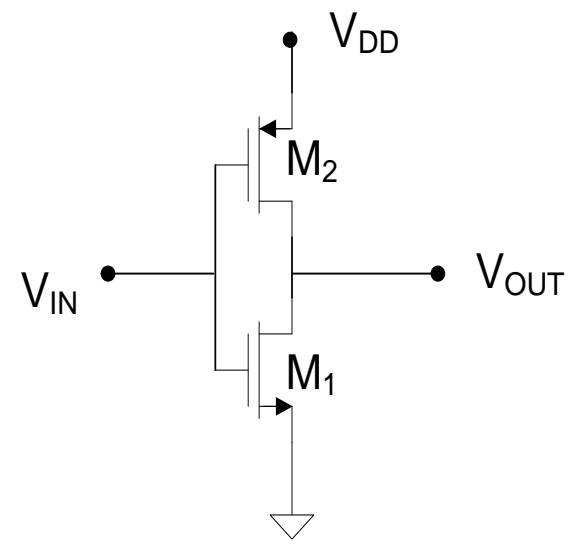


Consider first the inverter



# Transfer characteristics of the static CMOS inverter







# Transfer characteristics of the static CMOS inverter

(Neglect  $\lambda$  effects)

Case 1  $V_{IN}$  is so high that  $M_1$  triode,  $M_2$  cutoff

$$I_{D1} = \mu_n C_{oxn} \frac{W_1}{L_1} \left( V_{IN} - V_{Tn} - \frac{V_{OUT}}{2} \right) V_{OUT}$$

$$I_{D2} = 0$$

Equating  $I_{D1}$  and  $-I_{D2}$  we

obtain:

$$0 = \mu_n C_{oxn} \frac{W_1}{L_1} \left( V_{IN} - V_{Tn} - \frac{V_{OUT}}{2} \right) V_{OUT}$$

It can be shown that setting the first product term to 0 will not verify, thus

$$V_{OUT} = 0$$

valid for:

$$V_{GS1} \geq V_{Tn}$$

$$V_{DS1} < V_{GS1} - V_{Tn}$$

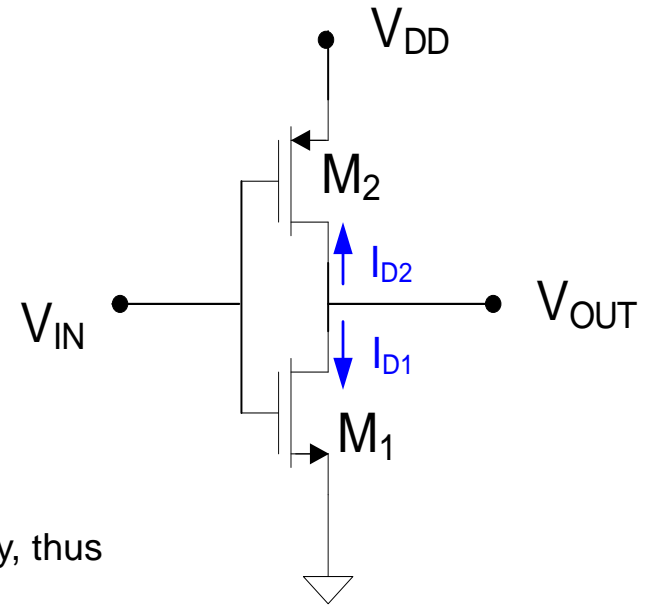
$$V_{GS2} \geq V_{Tp}$$

thus, valid for:

$$V_{IN} \geq V_{Tn}$$

$$V_{OUT} < V_{IN} - V_{Tn}$$

$$V_{IN} - V_{DD} \geq V_{Tp}$$



# Graphical Interpretation of these conditions:

$$V_{IN} \geq V_{Tn}$$

$$V_{OUT} < V_{IN} - V_{Tn}$$

$$V_{IN} - V_{DD} \geq V_{Tp}$$

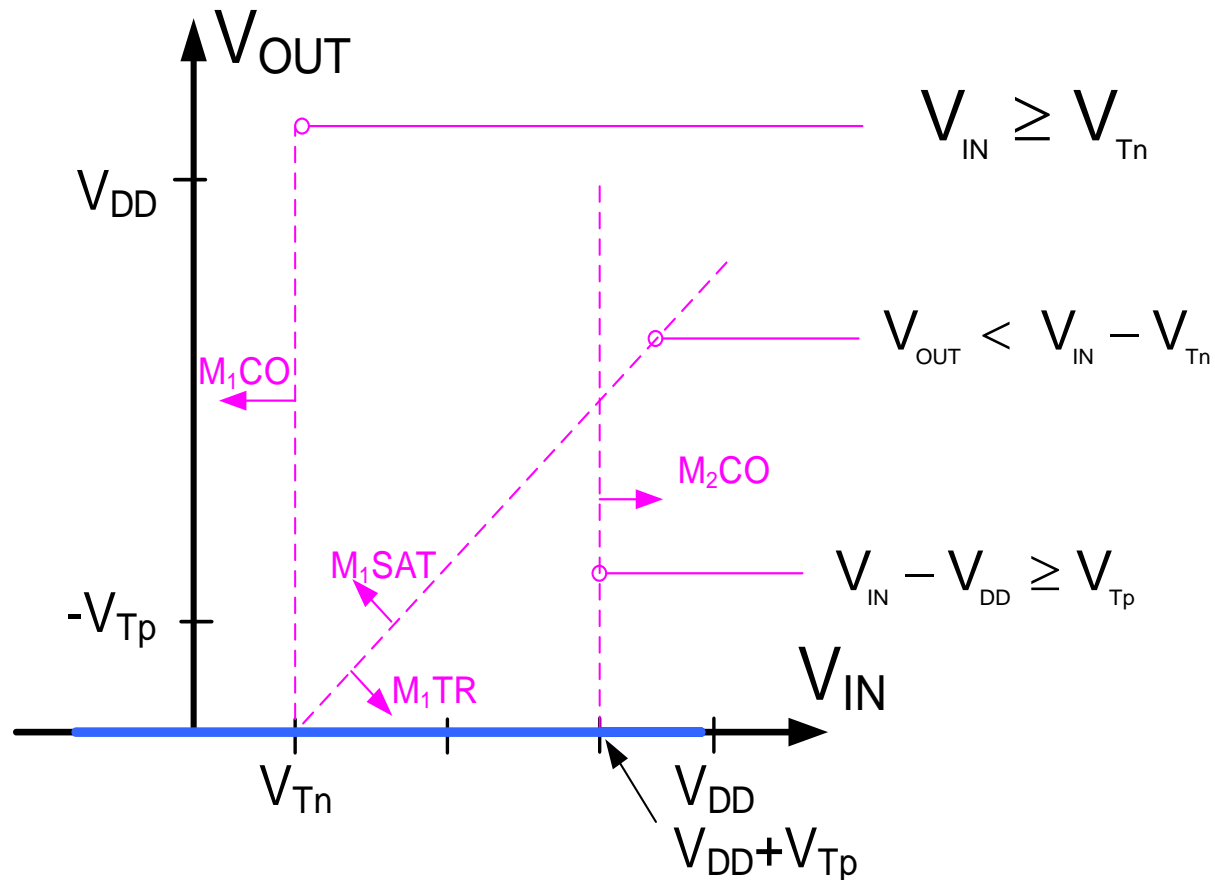


# Transfer characteristics of the static CMOS inverter

(Neglect  $\lambda$  effects)

Case 1  $M_1$  triode,  $M_2$  cutoff

$$V_{OUT} = 0$$

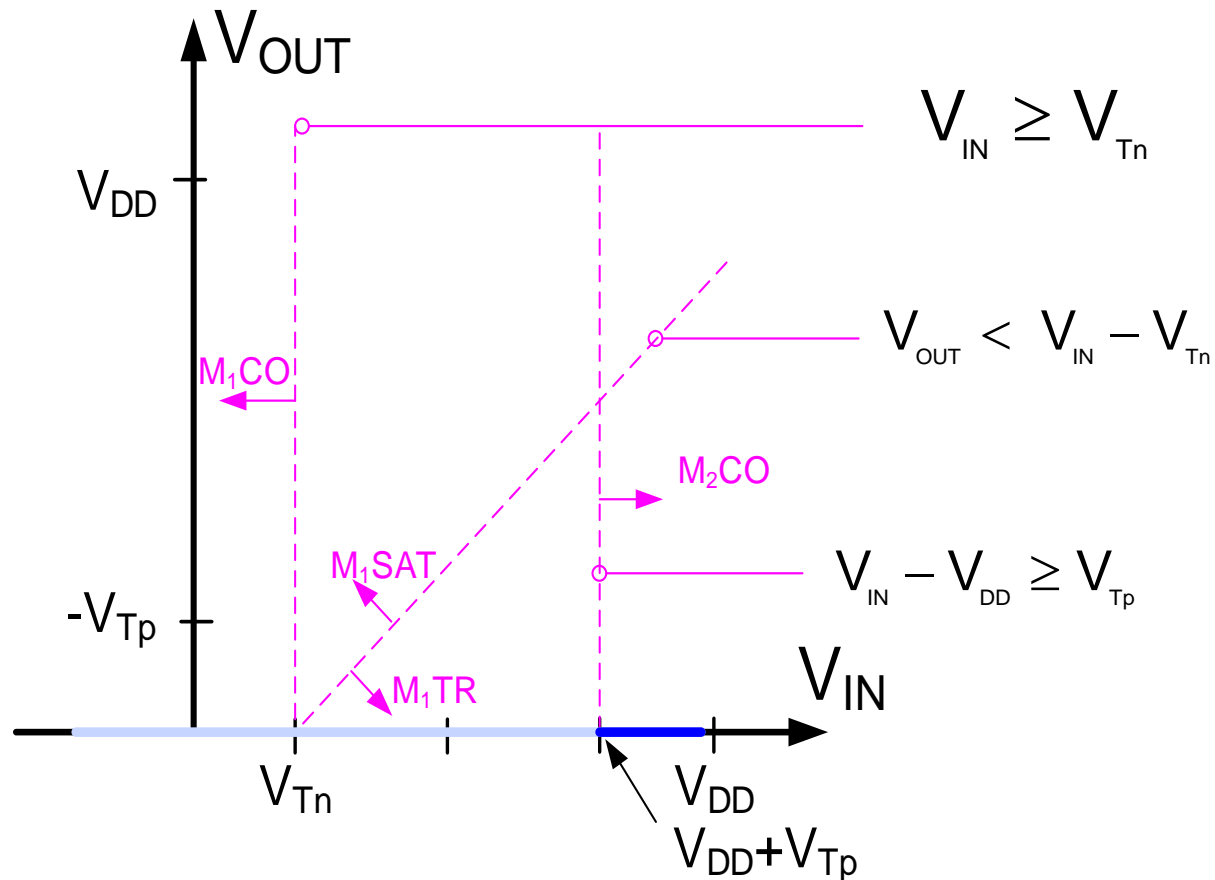


# Transfer characteristics of the static CMOS inverter

(Neglect  $\lambda$  effects)

Case 1  $M_1$  triode,  $M_2$  cutoff

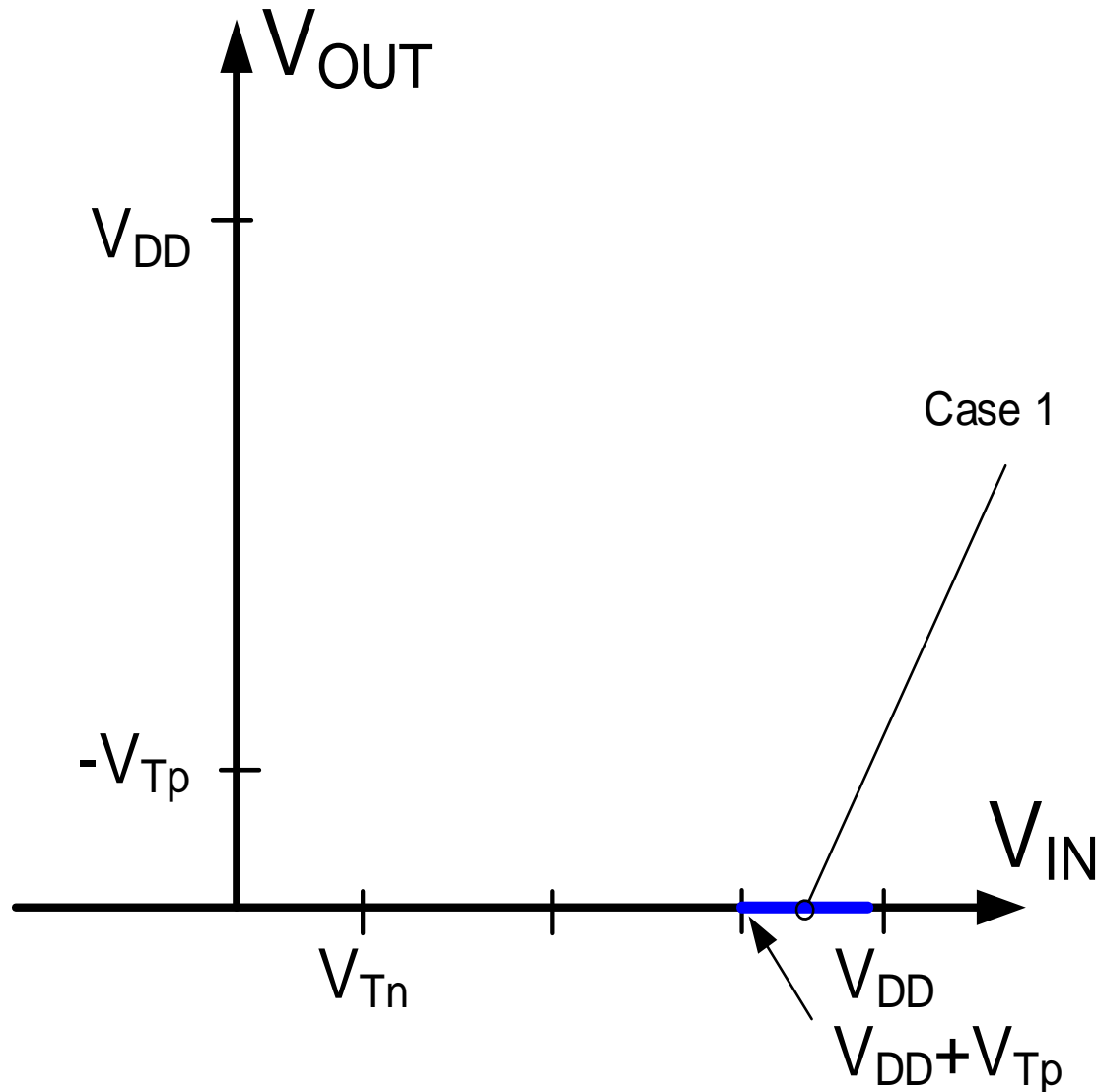
$$V_{OUT} = 0$$



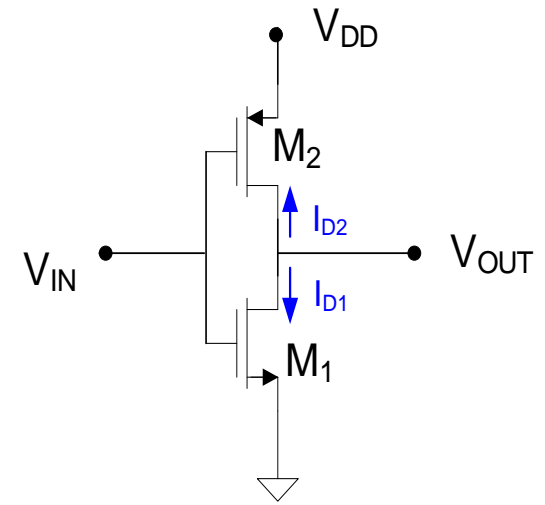
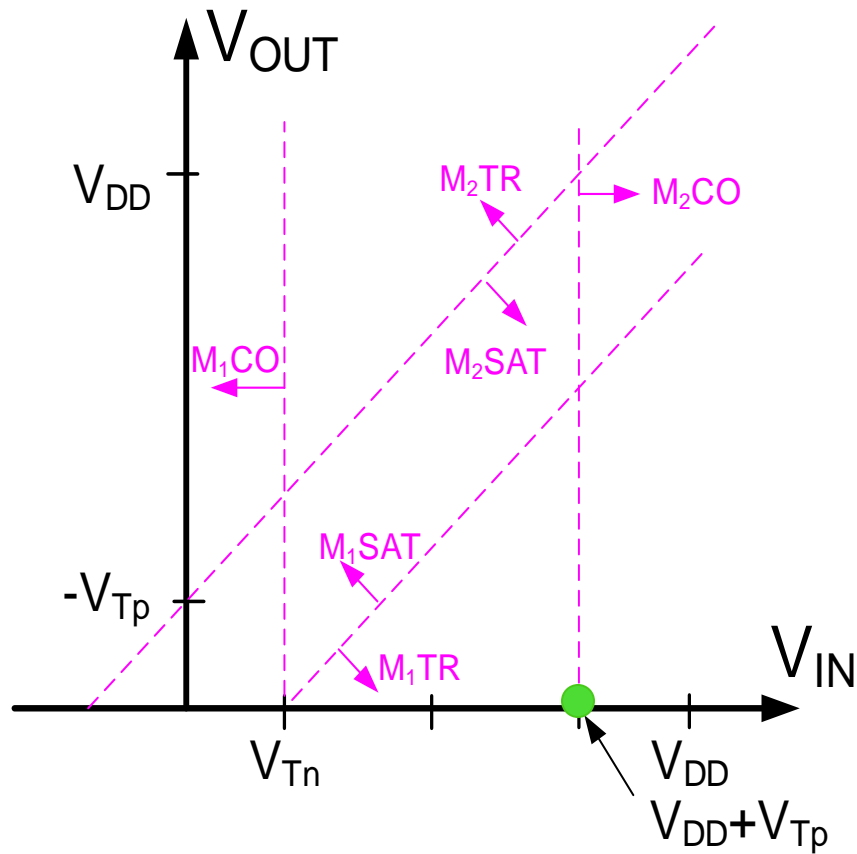
# Transfer characteristics of the static CMOS inverter

(Neglect  $\lambda$  effects)

Partial solution:



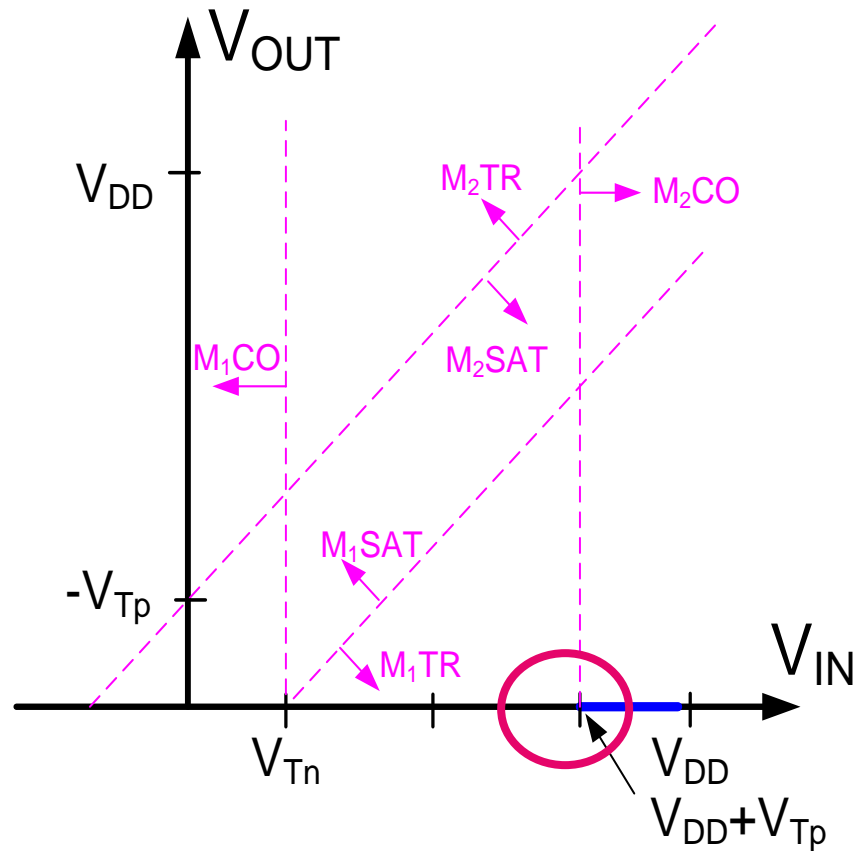
# Regions of Operation for Devices in CMOS inverter



# Transfer characteristics of the static CMOS inverter

(Neglect  $\lambda$  effects)

Case 2  $M_1$  triode,  $M_2$  sat



$M_2$ : Square law  $I_D$

$M_1$ : like a resistor

# Transfer characteristics of the static CMOS inverter

(Neglect  $\lambda$  effects)

Case 2  $M_1$  triode,  $M_2$  sat

$$I_{D1} = \mu_n C_{oxn} \frac{W_1}{L_1} \left( V_{IN} - V_{Tn} - \frac{V_{OUT}}{2} \right) V_{OUT}$$

$$I_{D2} = -\frac{\mu_p C_{oxp}}{2} \frac{W_2}{L_2} (V_{IN} - V_{DD} - V_{Tp})^2$$

Equating  $I_{D1}$  and  $-I_{D2}$  we obtain:

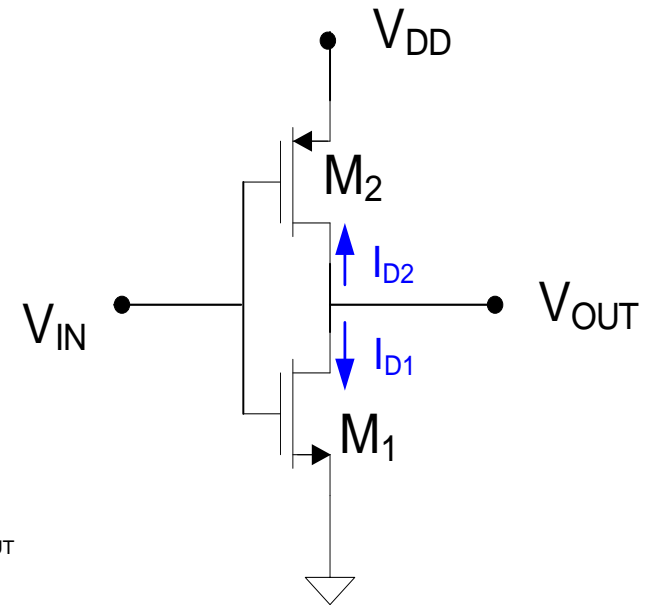
$$\frac{\mu_p C_{oxp}}{2} \frac{W_2}{L_2} (V_{IN} - V_{DD} - V_{Tp})^2 = \mu_n C_{oxn} \frac{W_1}{L_1} \left( V_{IN} - V_{Tn} - \frac{V_{OUT}}{2} \right) V_{OUT}$$

valid for:

$$V_{GS1} \geq V_{Tn} \quad V_{DS1} < V_{GS1} - V_{Tn} \quad V_{GS2} \leq V_{Tp} \quad V_{DS2} \leq V_{GS2} - V_{T2}$$

thus, valid for:

$$V_{IN} \geq V_{Tn} \quad V_{OUT} < V_{IN} - V_{Tn} \quad V_{IN} - V_{DD} \leq V_{Tp} \quad V_{OUT} - V_{DD} \leq V_{IN} - V_{DD} - V_{Tp}$$

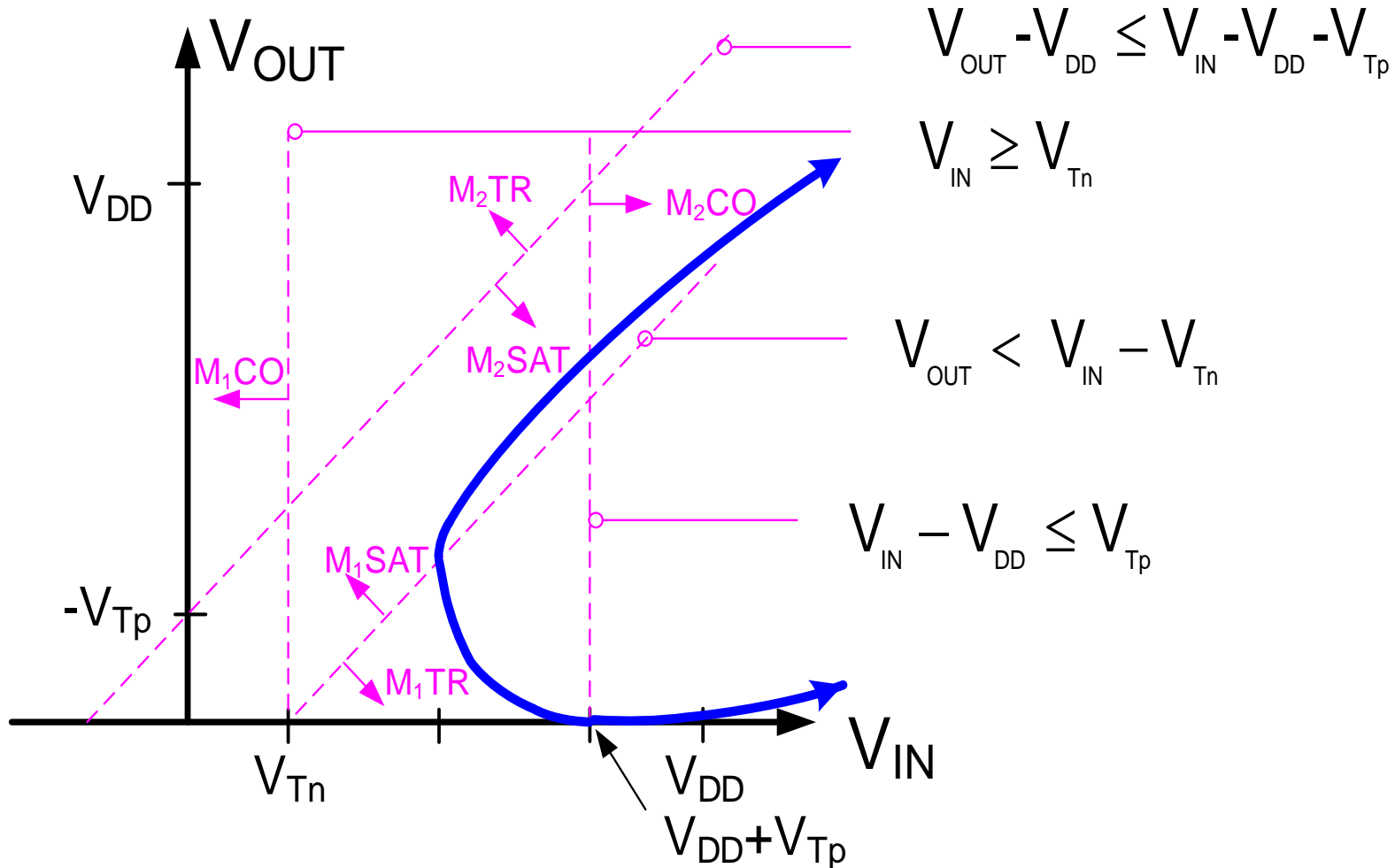




# Transfer characteristics of the static CMOS inverter

(Neglect  $\lambda$  effects)

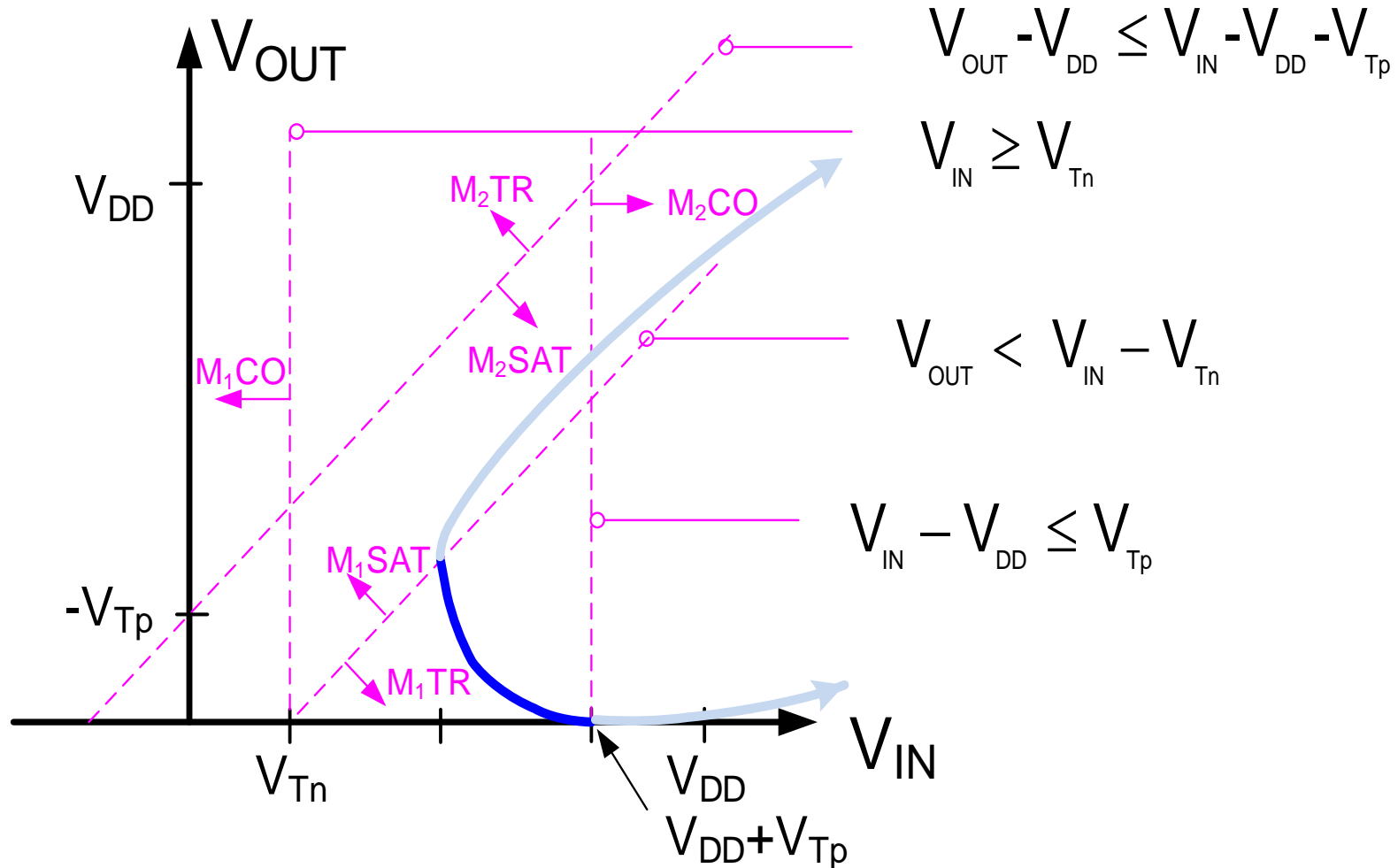
Case 2  $M_1$  triode,  $M_2$  sat



# Transfer characteristics of the static CMOS inverter

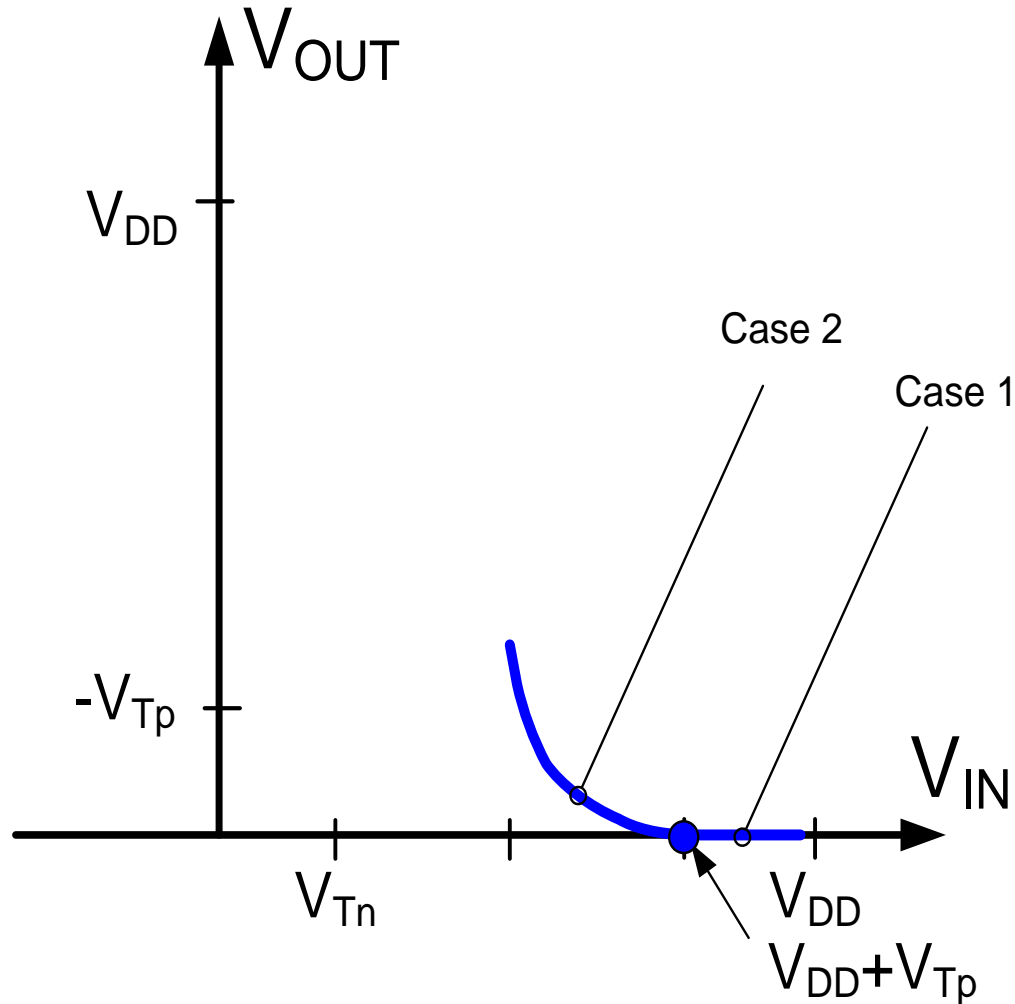
(Neglect  $\lambda$  effects)

Case 2  $M_1$  triode,  $M_2$  sat



# Transfer characteristics of the static CMOS inverter

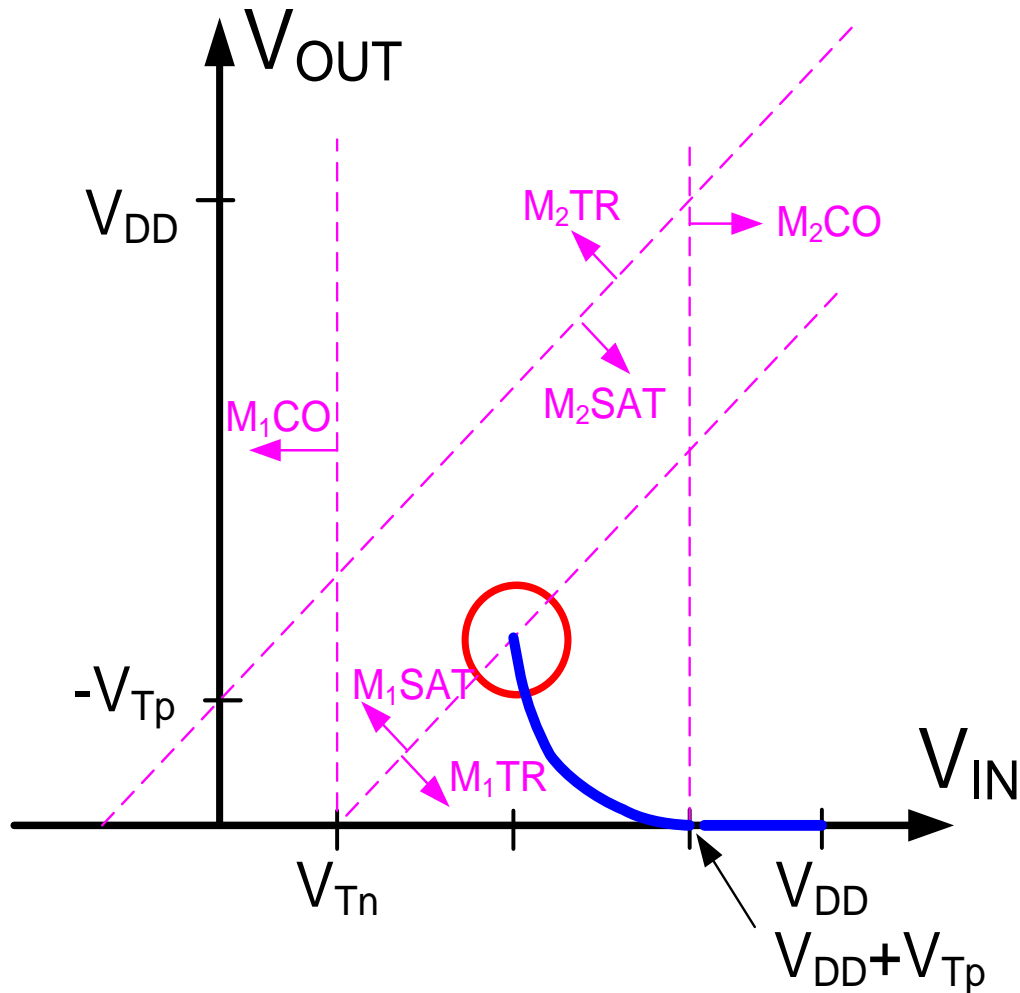
Partial solution:



# Transfer characteristics of the static CMOS inverter

(Neglect  $\lambda$  effects)

Case 3  $M_1$  sat,  $M_2$  sat



# Transfer characteristics of the static CMOS inverter

(Neglect  $\lambda$  effects)

Case 3  $M_1$  sat,  $M_2$  sat

$$I_{D1} = \frac{\mu_n C_{OXn}}{2} \frac{W_1}{L_1} (V_{IN} - V_{Tn})^2$$

$$I_{D2} = \frac{\mu_p C_{OXp}}{2} \frac{W_2}{L_2} (V_{IN} - V_{DD} - V_{Tp})^2$$

Equating  $I_{D1}$  and  $-I_{D2}$  we obtain:

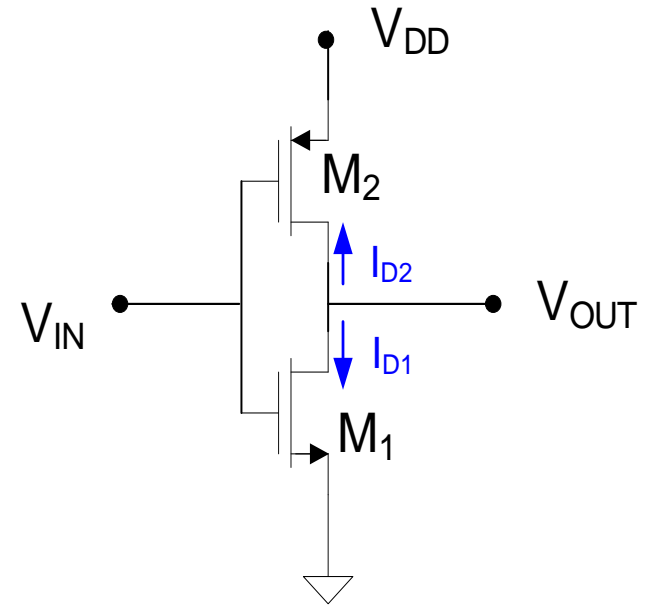
$$\frac{\mu_p C_{OXp}}{2} \frac{W_2}{L_2} (V_{IN} - V_{DD} - V_{Tp})^2 = \frac{\mu_n C_{OXn}}{2} \frac{W_1}{L_1} (V_{IN} - V_{Tn})^2$$

Which can be rewritten as:

$$\sqrt{\frac{\mu_p C_{OXp}}{2} \frac{W_2}{L_2}} (V_{DD} + V_{Tp} - V_{IN}) = \sqrt{\frac{\mu_n C_{OXn}}{2} \frac{W_1}{L_1}} (V_{IN} - V_{Tn})$$

Which can be simplified to:

$$V_{IN} = \frac{(V_{Tn}) \sqrt{\frac{\mu_n C_{OXn}}{2} \frac{W_1}{L_1}} + (V_{DD} + V_{Tp}) \sqrt{\frac{\mu_p C_{OXp}}{2} \frac{W_2}{L_2}}}{\sqrt{\frac{\mu_n C_{OXn}}{2} \frac{W_1}{L_1}} + \sqrt{\frac{\mu_p C_{OXp}}{2} \frac{W_2}{L_2}}}$$



This is a vertical line

# Transfer characteristics of the static CMOS inverter

(Neglect  $\lambda$  effects)

Case 3  $M_1$  sat,  $M_2$  sat

$$V_{IN} = \frac{(V_{Tn}) \sqrt{\frac{\mu_n C_{oxn}}{2} \frac{W_1}{L_1}} + (V_{DD} + V_{Tp}) \sqrt{\frac{\mu_p C_{oxp}}{2} \frac{W_2}{L_2}}}{\sqrt{\frac{\mu_n C_{oxn}}{2} \frac{W_1}{L_1}} + \sqrt{\frac{\mu_p C_{oxp}}{2} \frac{W_2}{L_2}}}$$

Since  $C_{oxn} \cong C_{oxp} = C_{ox}$  this can be simplified to:

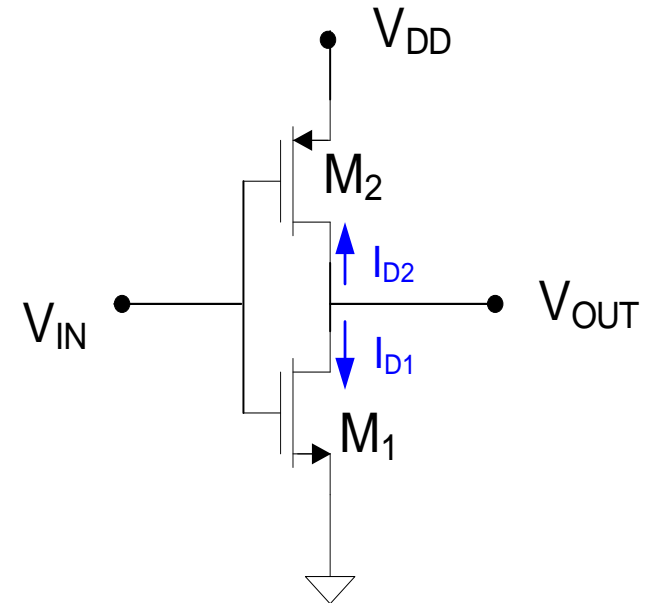
$$V_{IN} = \frac{(V_{Tn}) \sqrt{\frac{W_1}{L_1}} + (V_{DD} + V_{Tp}) \sqrt{\frac{\mu_p}{\mu_n} \frac{W_2}{L_2}}}{\sqrt{\frac{W_1}{L_1}} + \sqrt{\frac{\mu_p}{\mu_n} \frac{W_2}{L_2}}}$$

valid for:

$$V_{GS1} \geq V_{Tn} \quad V_{DS1} \geq V_{GS1} - V_{Tn} \quad V_{GS2} \leq V_{Tp} \quad V_{DS2} \leq V_{GS2} - V_{T2}$$

thus, valid for:

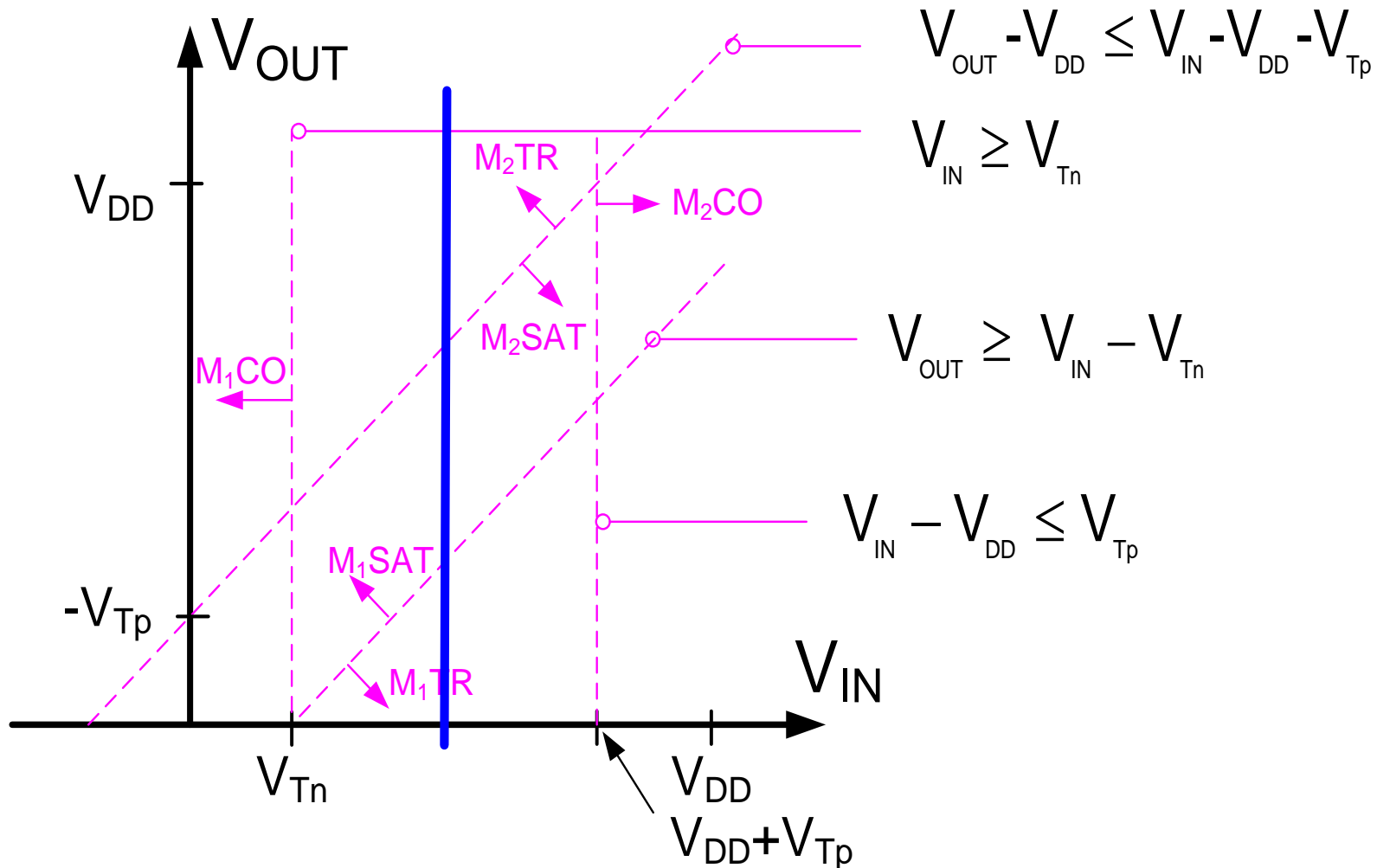
$$V_{IN} \geq V_{Tn} \quad V_{OUT} \geq V_{IN} - V_{Tn} \quad V_{IN} - V_{DD} \leq V_{Tp} \quad V_{OUT} - V_{DD} \leq V_{IN} - V_{DD} - V_{Tp}$$



# Transfer characteristics of the static CMOS inverter

(Neglect  $\lambda$  effects)

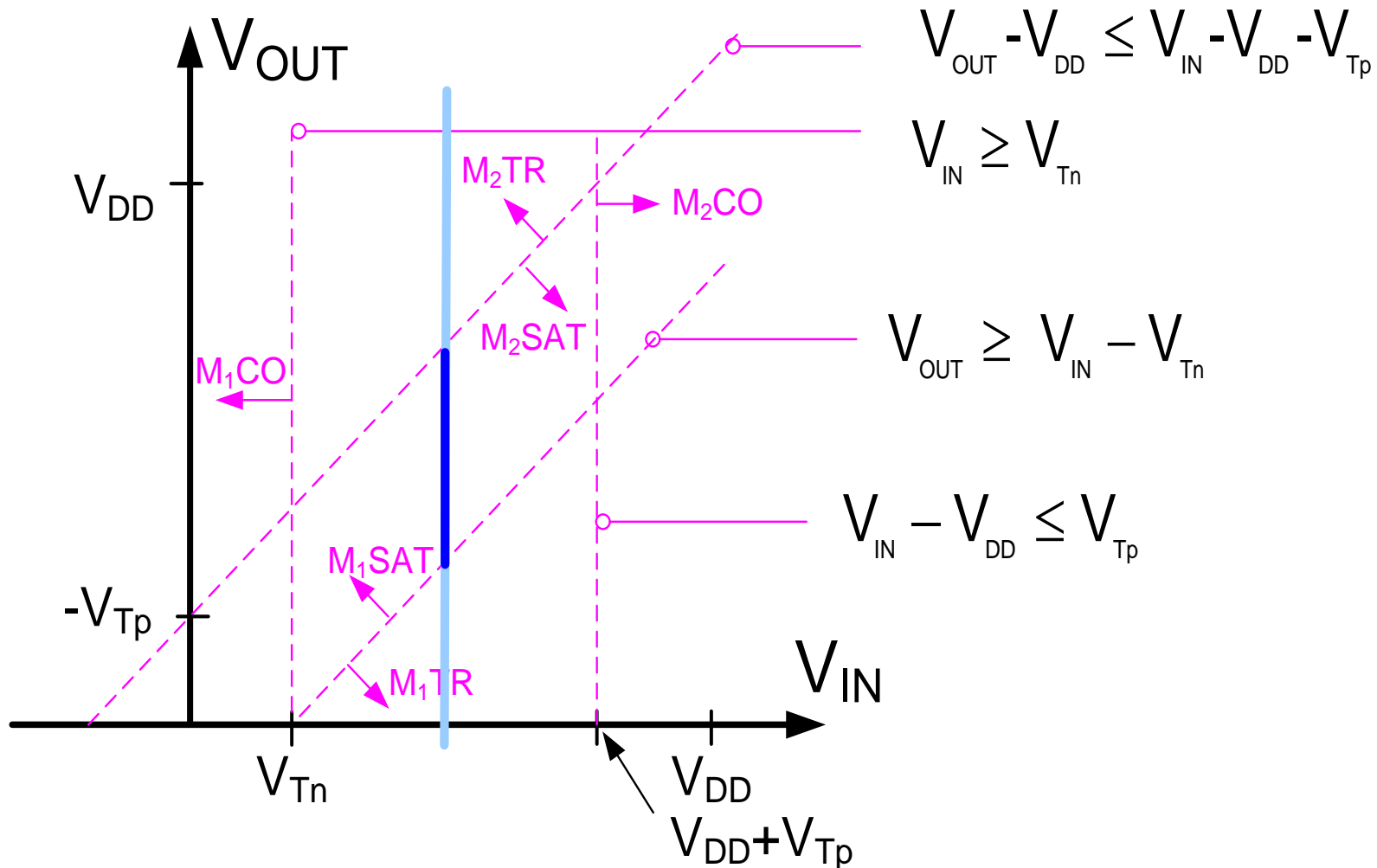
Case 3  $M_1$  sat,  $M_2$  sat



# Transfer characteristics of the static CMOS inverter

(Neglect  $\lambda$  effects)

Case 3  $M_1$  sat,  $M_2$  sat

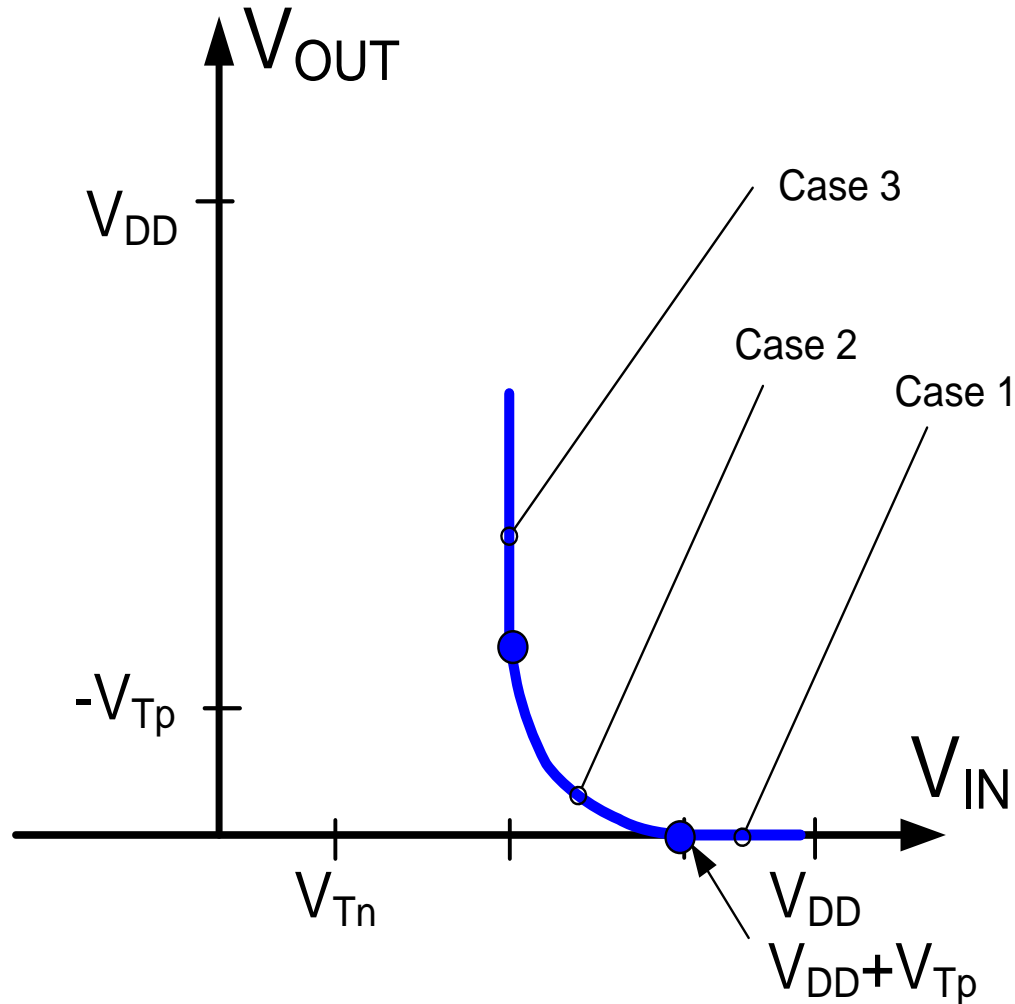




# Transfer characteristics of the static CMOS inverter

(Neglect  $\lambda$  effects)

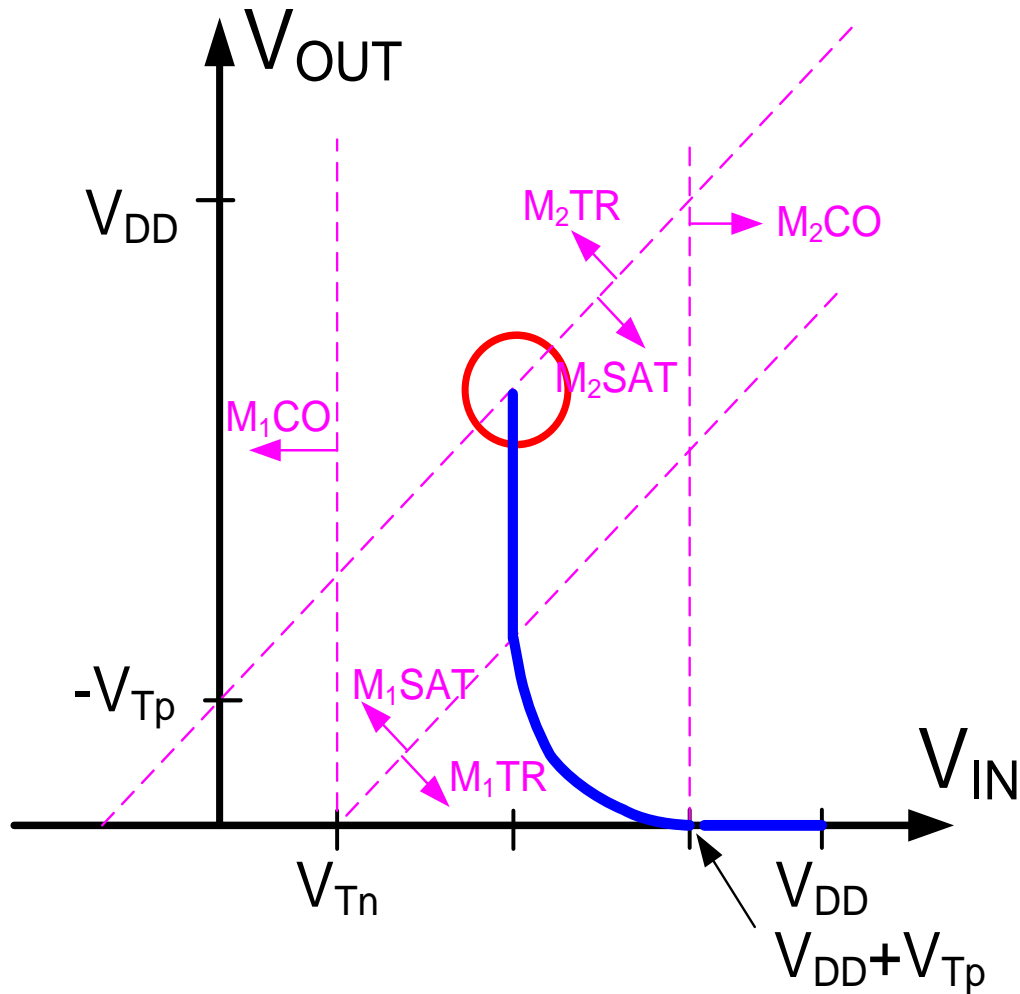
Partial solution:



# Transfer characteristics of the static CMOS inverter

(Neglect  $\lambda$  effects)

Case 4  $M_1$  sat,  $M_2$  triode



# Transfer characteristics of the static CMOS inverter

(Neglect  $\lambda$  effects)

Case 4  $M_1$  sat,  $M_2$  triode

$$I_{D1} = \frac{\mu_n C_{oxn}}{2} \frac{W_1}{L_1} (V_{IN} - V_{Tn})^2$$

$$I_{D2} = -\mu_p C_{oxp} \frac{W_2}{L_2} \left( V_{IN} - V_{DD} - V_{Tp} - \frac{V_{OUT} - V_{DD}}{2} \right) \cdot (V_{OUT} - V_{DD})$$

Equating  $I_{D1}$  and  $-I_{D2}$  we obtain:

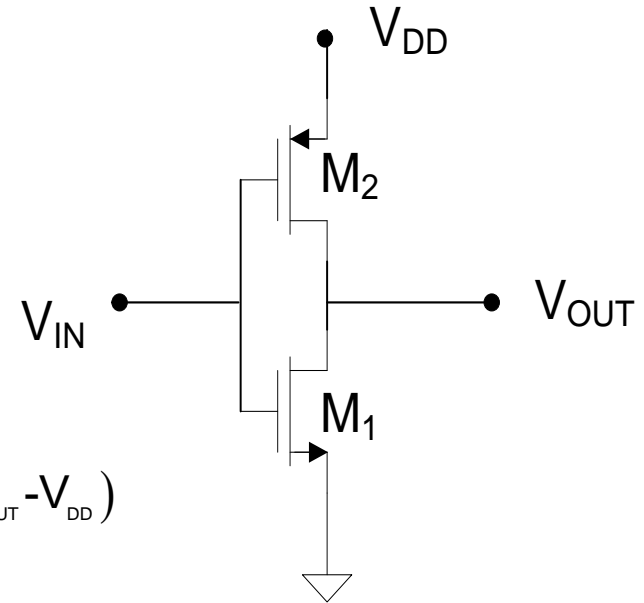
$$\frac{\mu_n C_{oxn}}{2} \frac{W_1}{L_1} (V_{IN} - V_{Tn})^2 = \mu_p C_{oxp} \frac{W_2}{L_2} \left( V_{IN} - V_{DD} - V_{Tp} - \frac{V_{OUT} - V_{DD}}{2} \right) \cdot (V_{OUT} - V_{DD})$$

valid for:

$$V_{GS1} \geq V_{Tn} \quad V_{DS1} \geq V_{GS1} - V_{Tn} \quad V_{GS2} \leq V_{Tp} \quad V_{DS2} > V_{GS2} - V_{Tp}$$

thus, valid for:

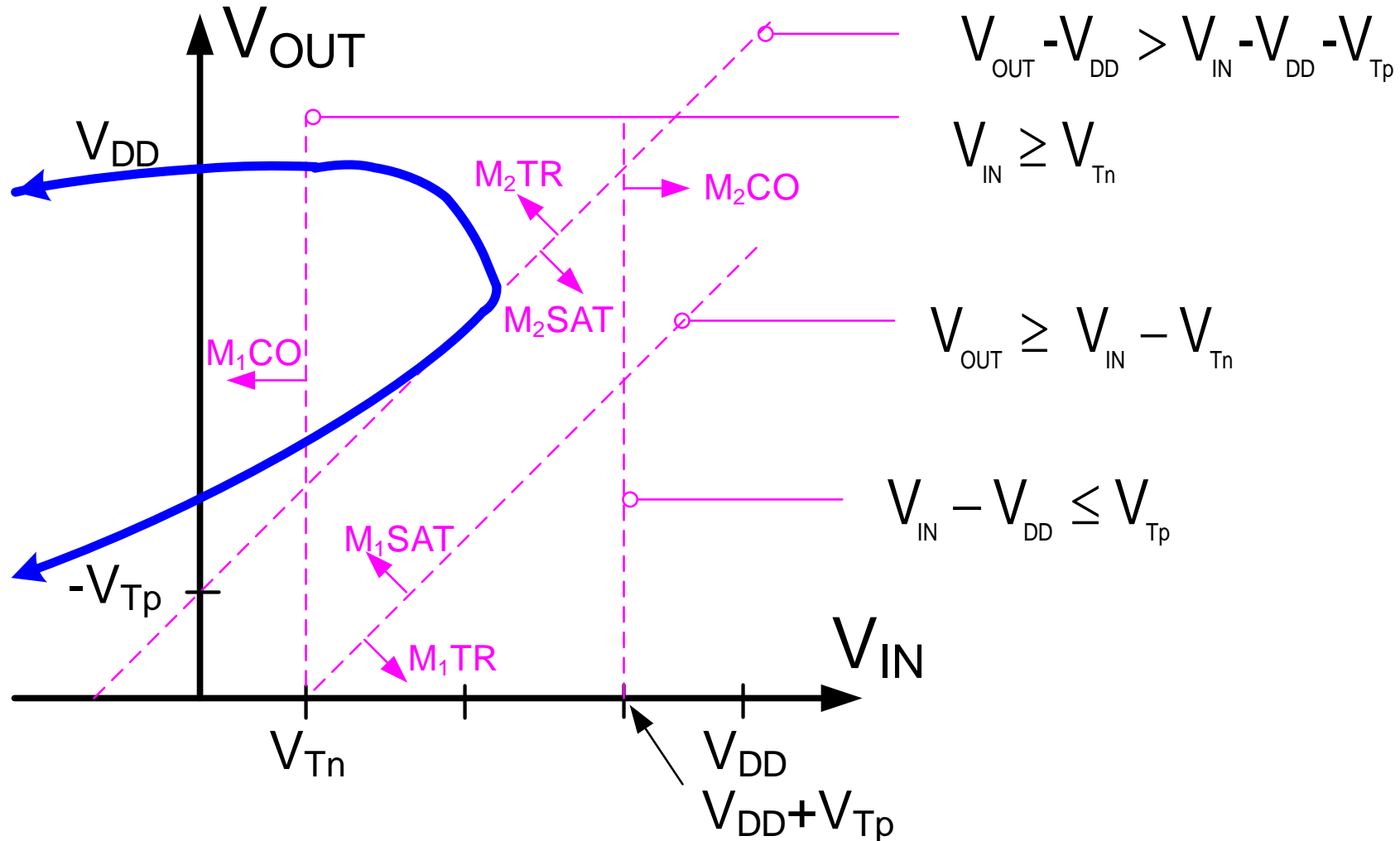
$$V_{IN} \geq V_{Tn} \quad V_{OUT} \geq V_{IN} - V_{Tn} \quad V_{IN} - V_{DD} \leq V_{Tp} \quad V_{OUT} - V_{DD} > V_{IN} - V_{DD} - V_{Tp}$$



# Transfer characteristics of the static CMOS inverter

(Neglect  $\lambda$  effects)

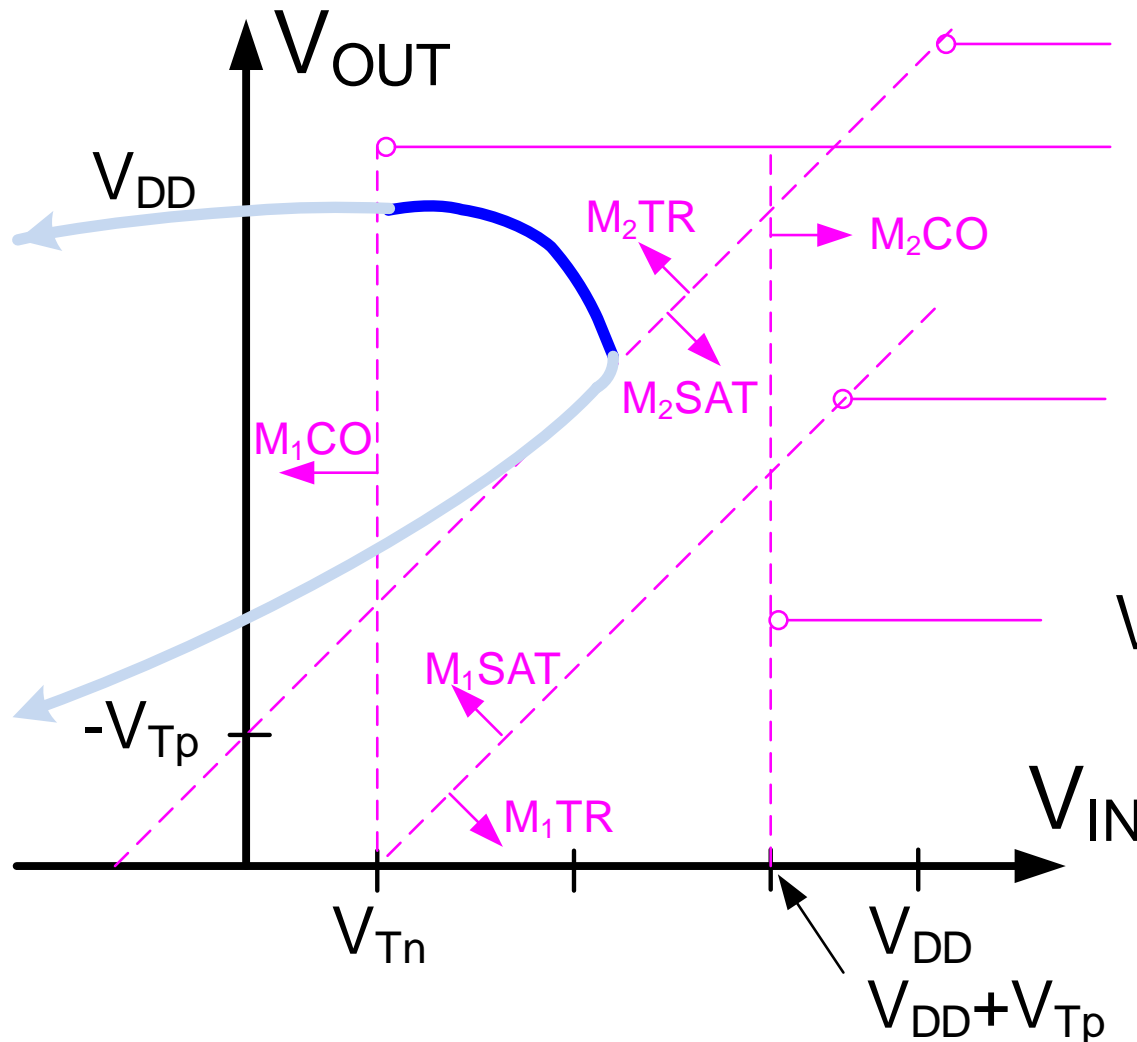
Case 4  $M_1$  sat,  $M_2$  triode



# Transfer characteristics of the static CMOS inverter

(Neglect  $\lambda$  effects)

Case 4  $M_1$  sat,  $M_2$  triode



$$V_{OUT} - V_{DD} > V_{IN} - V_{DD} - V_{Tp}$$

$$V_{IN} \geq V_{Tn}$$

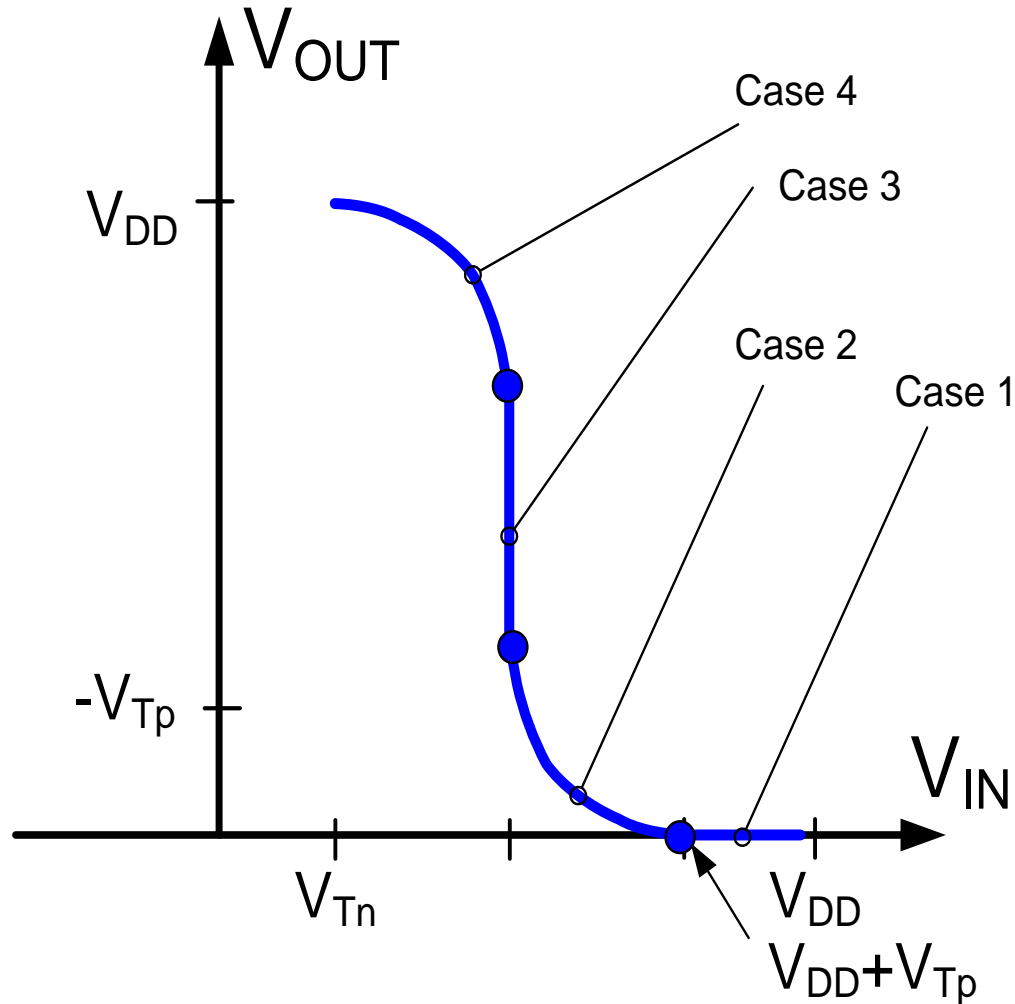
$$V_{OUT} \geq V_{IN} - V_{Tn}$$

$$V_{IN} - V_{DD} \leq V_{Tp}$$

# Transfer characteristics of the static CMOS inverter

(Neglect  $\lambda$  effects)

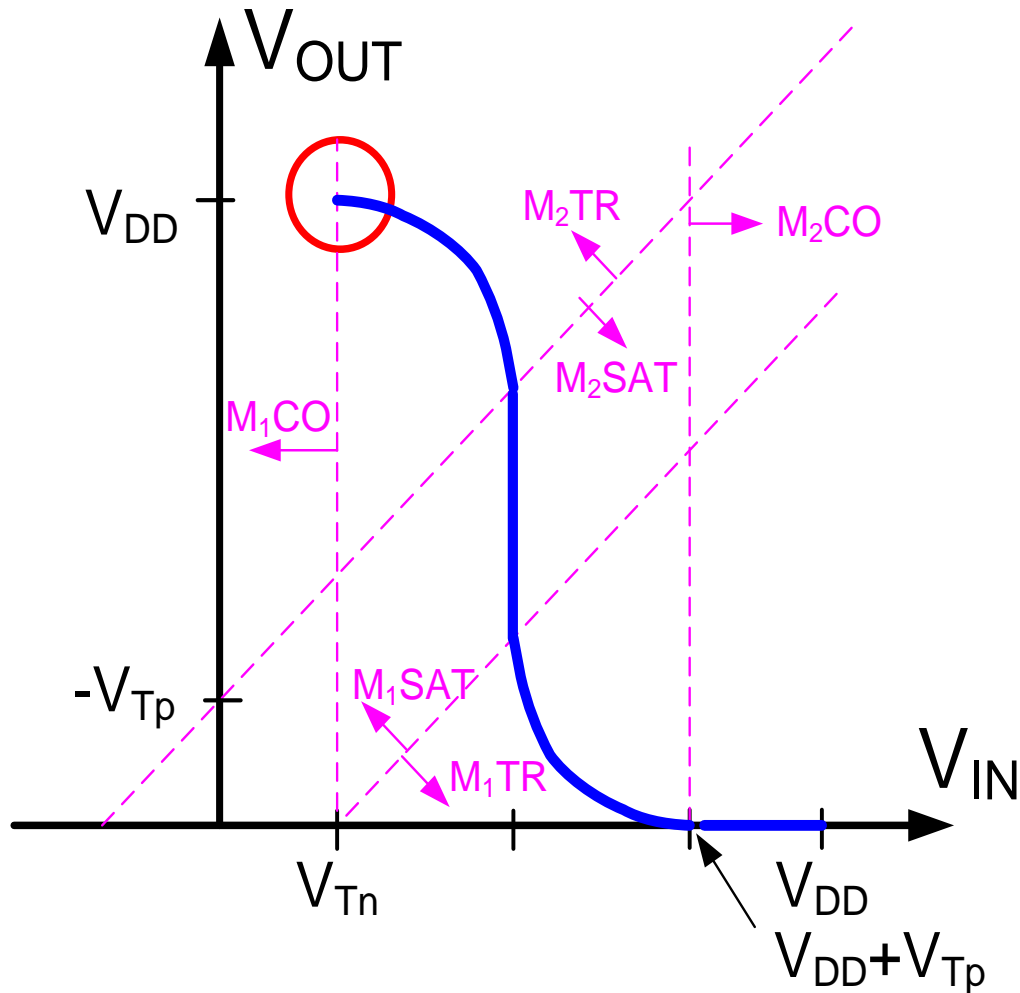
Partial solution:



# Transfer characteristics of the static CMOS inverter

(Neglect  $\lambda$  effects)

Case 4  $M_1$  cutoff,  $M_2$  triode



# Transfer characteristics of the static CMOS inverter

(Neglect  $\lambda$  effects)

Case 5  $M_1$  cutoff,  $M_2$  triode

$$I_{D1} = 0$$

$$I_{D2} = -\mu_p C_{oxp} \frac{W_2}{L_2} \left( V_{IN} - V_{DD} - V_{Tp} - \frac{V_{OUT} - V_{DD}}{2} \right) \cdot (V_{OUT} - V_{DD})$$

Equating  $I_{D1}$  and  $-I_{D2}$  we obtain:

$$\mu_p C_{oxp} \frac{W_2}{L_2} \left( V_{IN} - V_{DD} - V_{Tp} - \frac{V_{OUT} - V_{DD}}{2} \right) \cdot (V_{OUT} - V_{DD}) = 0$$

valid for:

$$V_{GS1} < V_{Tn}$$

$$V_{GS2} \leq V_{Tp}$$

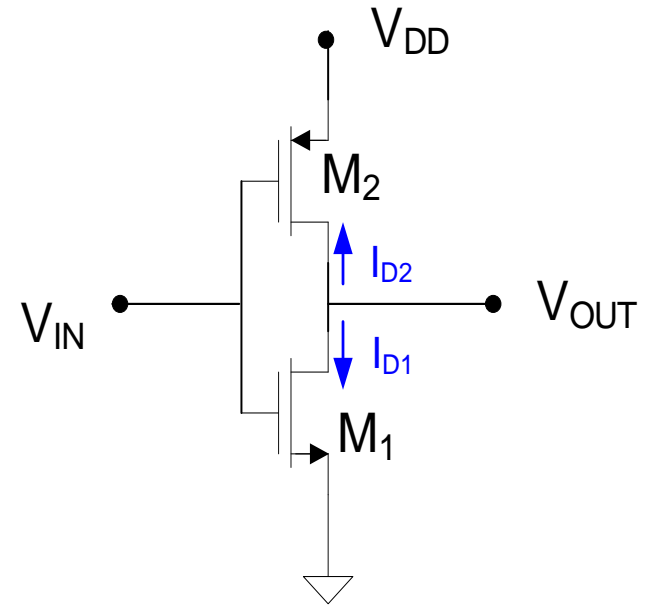
$$V_{DS2} > V_{GS2} - V_{T2}$$

thus, valid for:

$$V_{IN} < V_{Tn}$$

$$V_{IN} - V_{DD} \leq V_{Tp}$$

$$V_{OUT} - V_{DD} > V_{IN} - V_{DD} - V_{Tp}$$

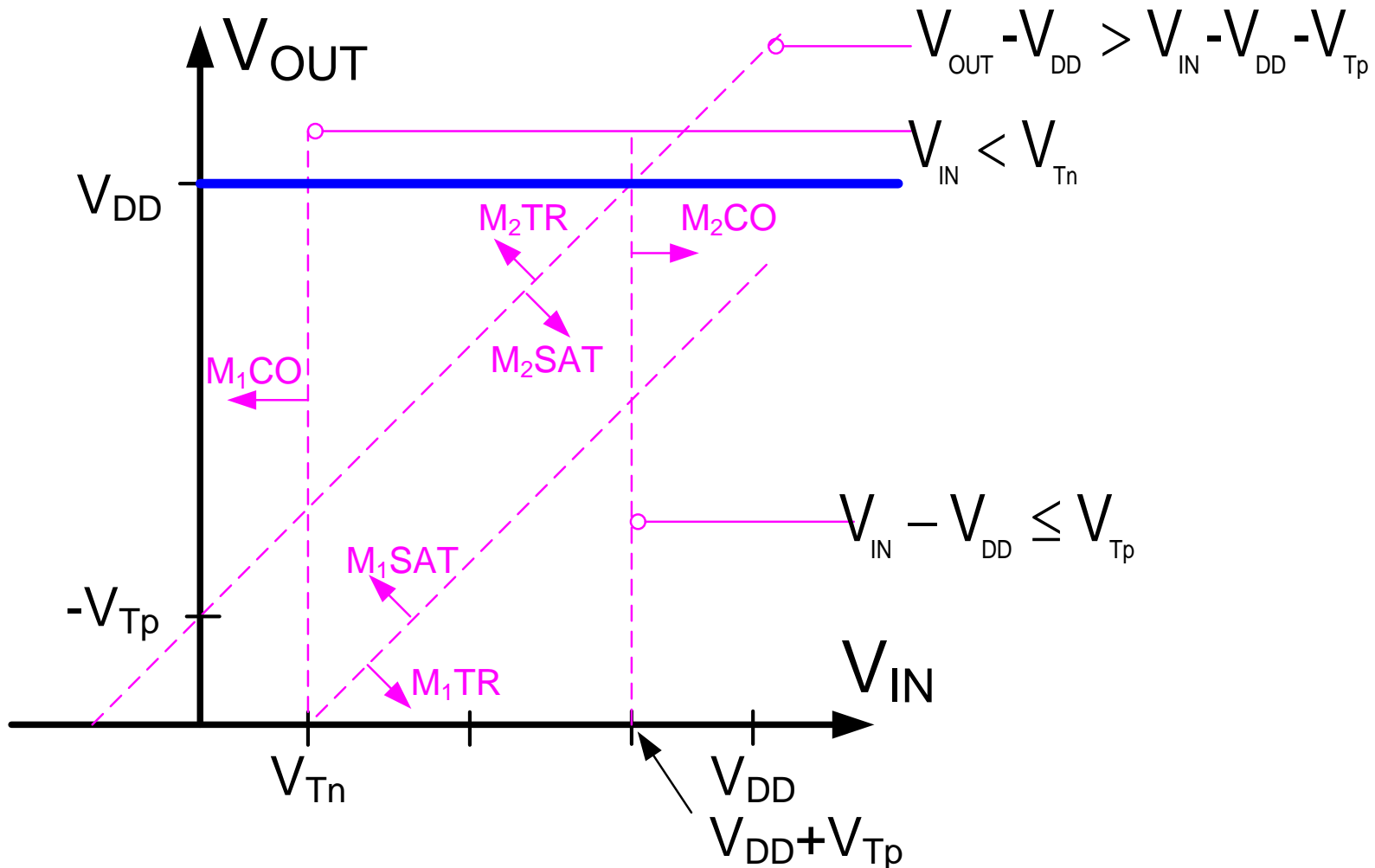




# Transfer characteristics of the static CMOS inverter

(Neglect  $\lambda$  effects)

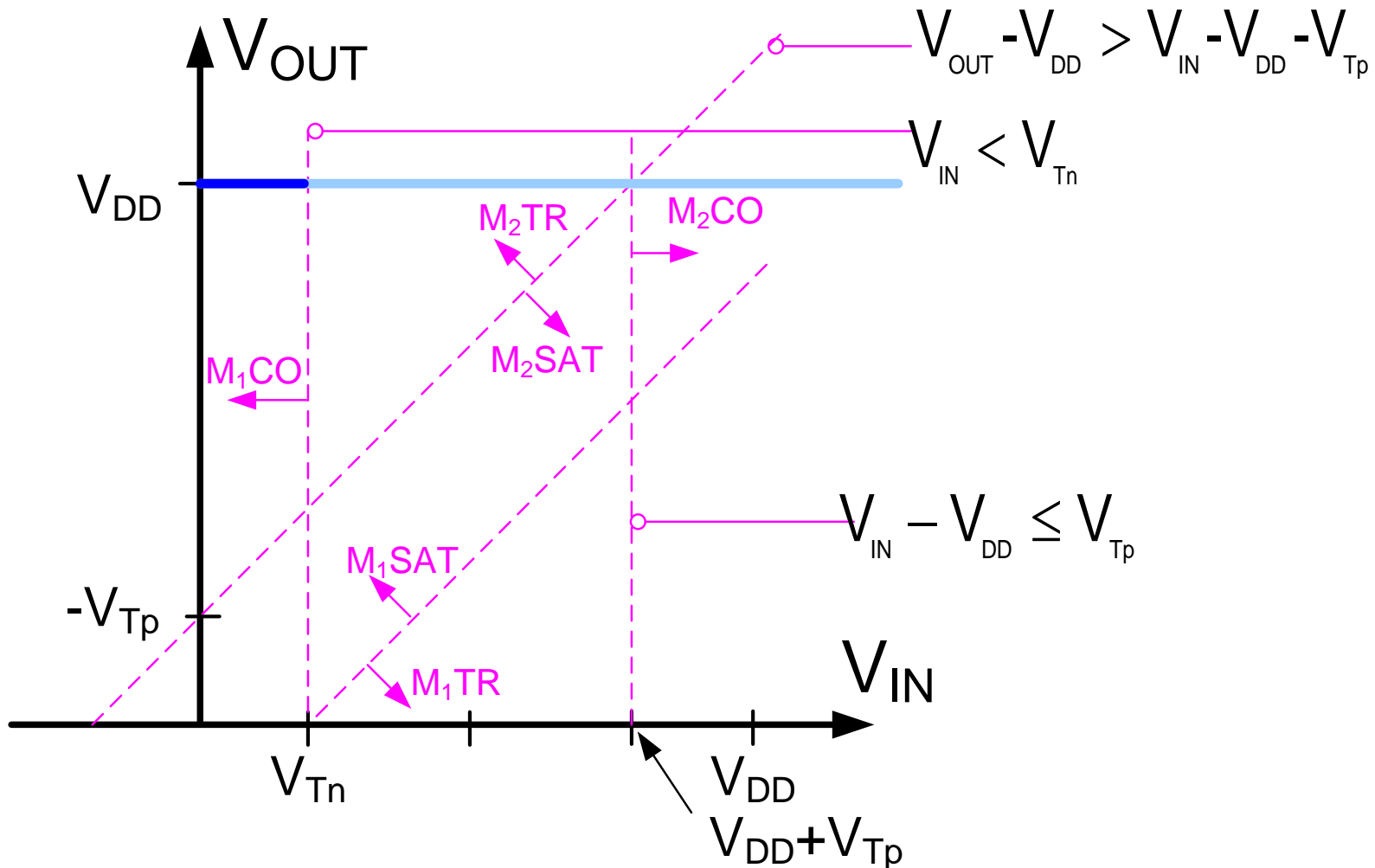
Case 5  $M_1$  cutoff,  $M_2$  triode



# Transfer characteristics of the static CMOS inverter

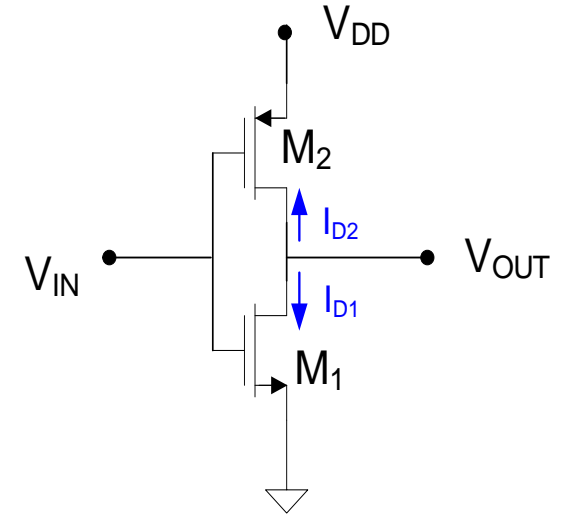
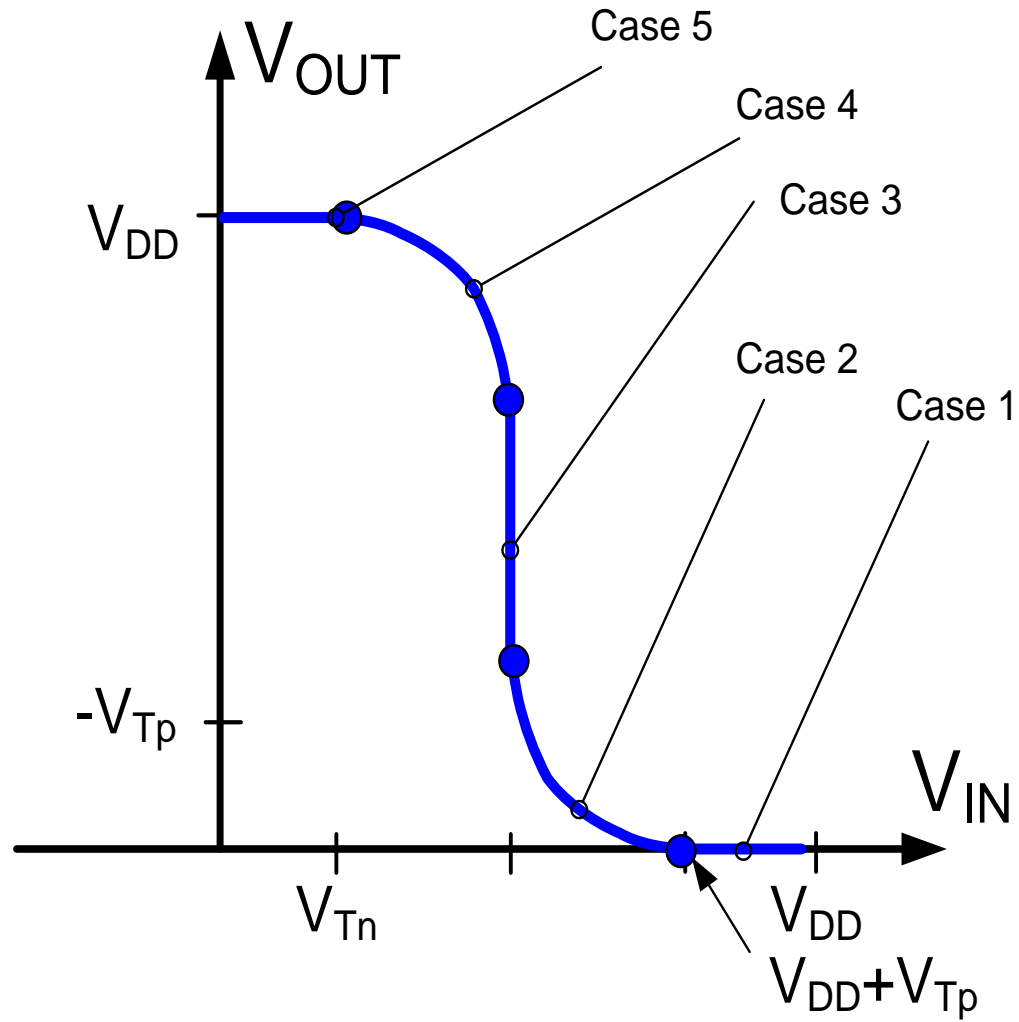
(Neglect  $\lambda$  effects)

Case 5  $M_1$  cutoff,  $M_2$  triode



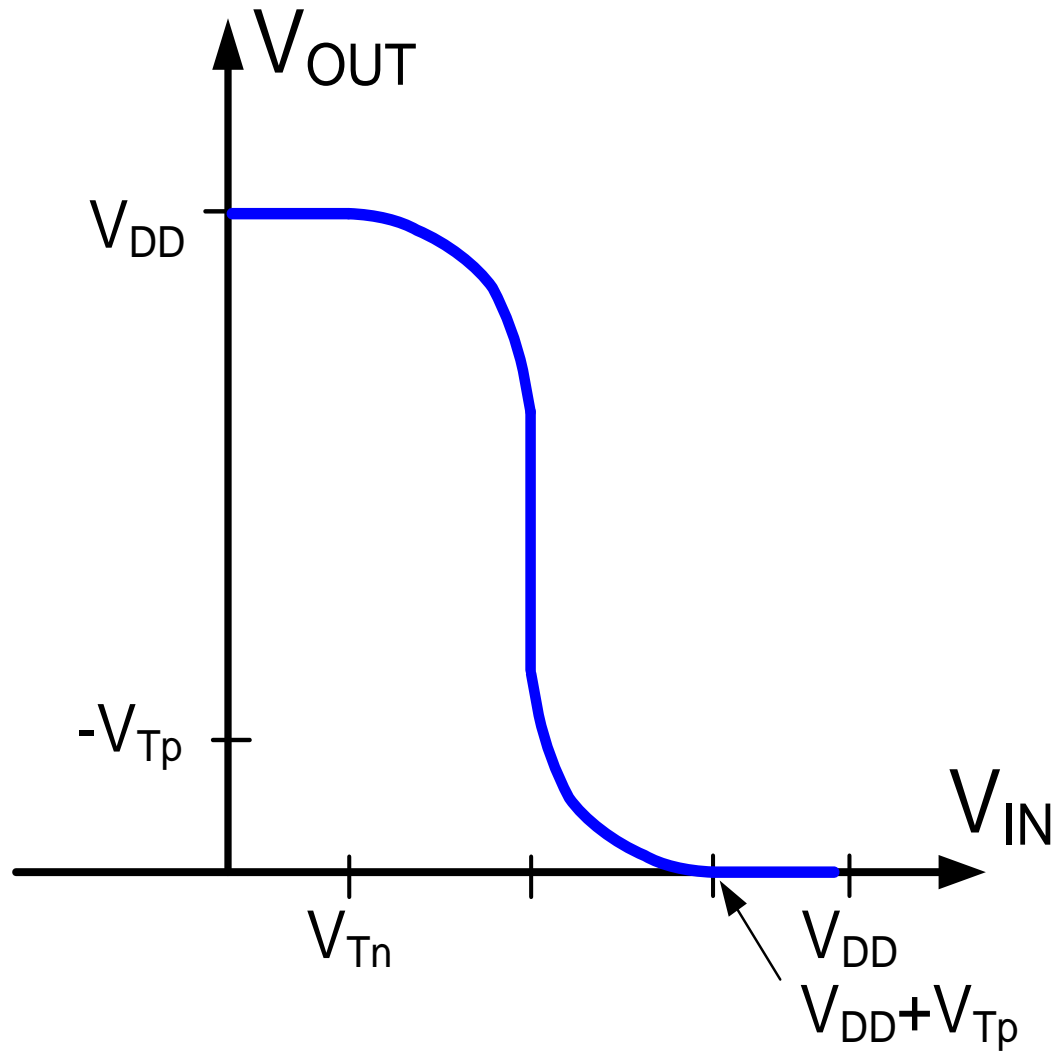
# Transfer characteristics of the static CMOS inverter

(Neglect  $\lambda$  effects)



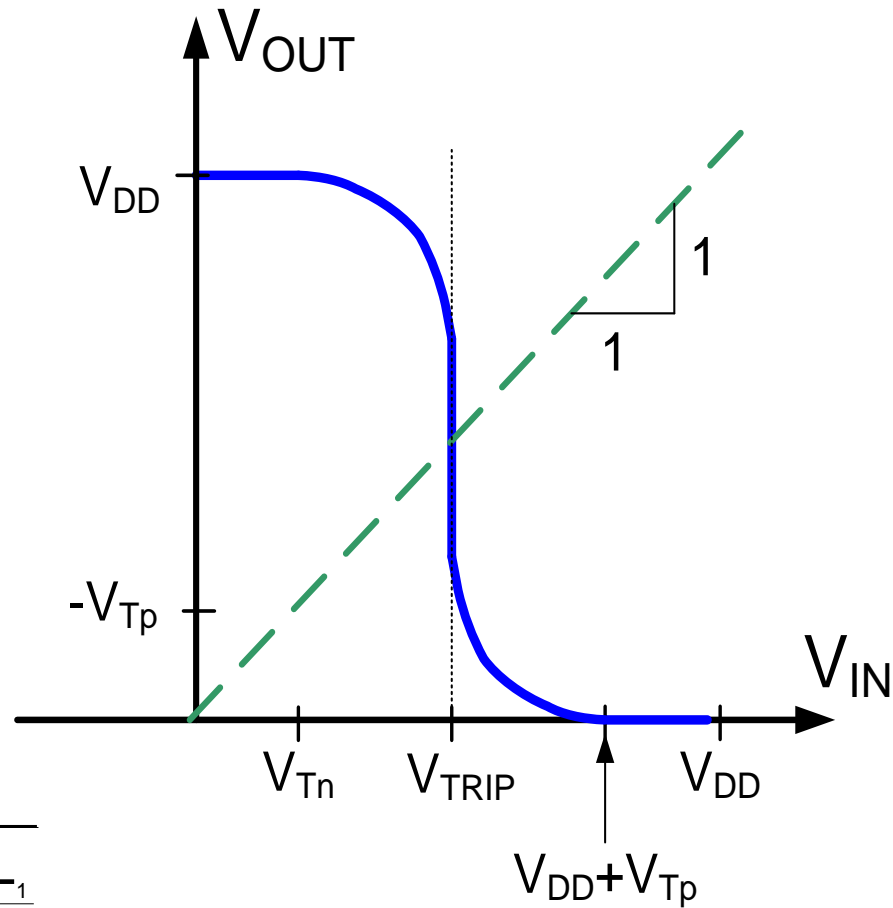
# Transfer characteristics of the static CMOS inverter

(Neglect  $\lambda$  effects)



# Transfer characteristics of the static CMOS inverter

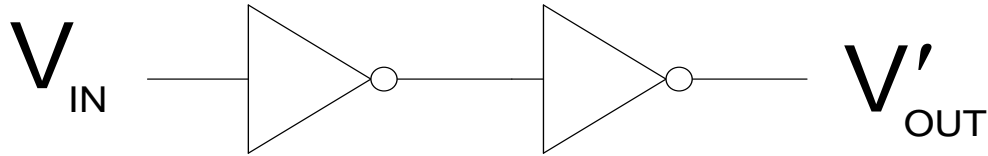
(Neglect  $\lambda$  effects)



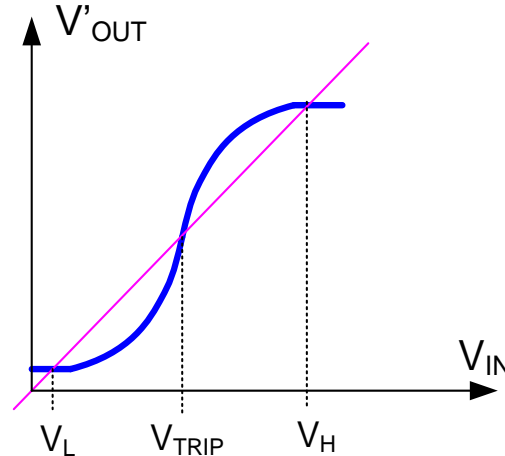
From Case 3 analysis:

$$V_{IN} = \frac{(V_{Tn}) + (V_{DD} + V_{Tp}) \sqrt{\frac{\mu_p}{\mu_n} \frac{W_2}{W_1} \frac{L_1}{L_2}}}{1 + \sqrt{\frac{\mu_p}{\mu_n} \frac{W_2}{W_1} \frac{L_1}{L_2}}}$$

# Inverter Transfer Characteristics of Inverter Pair

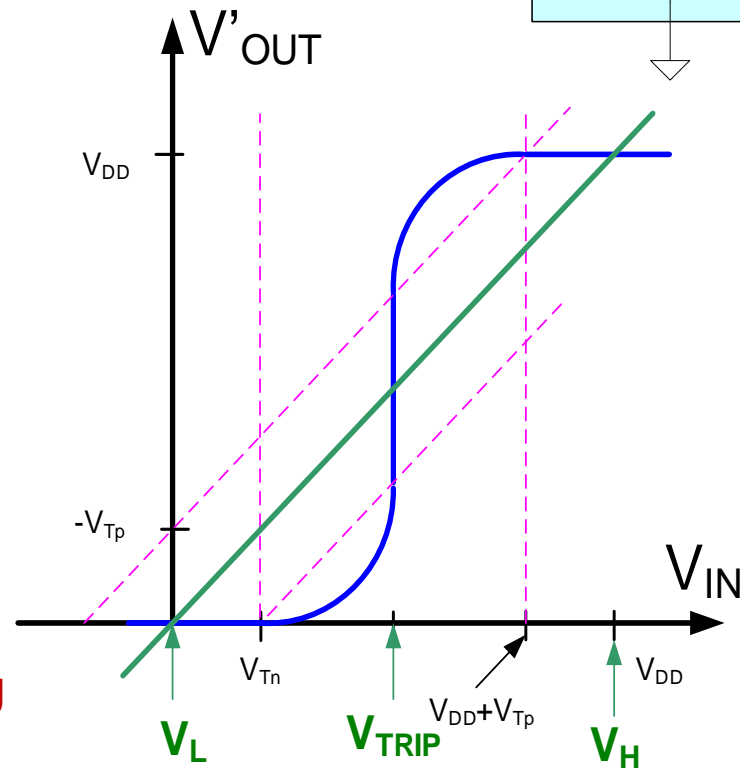
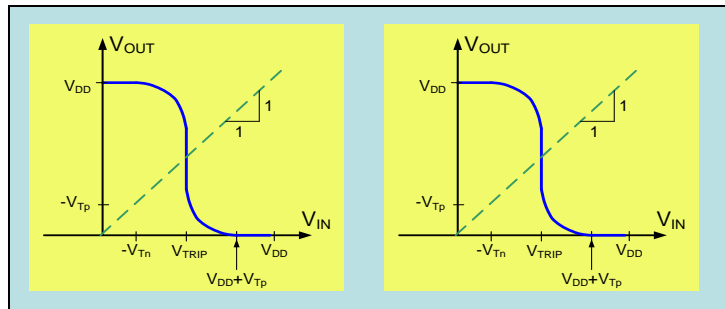
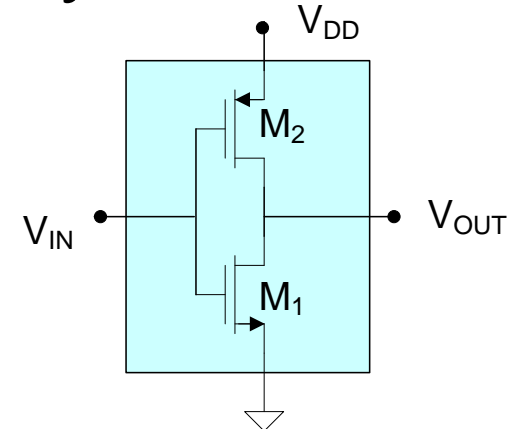
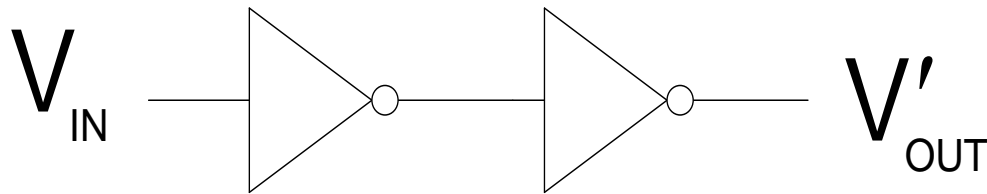


What are  $V_H$  and  $V_L$ ?



Find the points on the inverter pair transfer characteristics where  $V_{OUT}' = V_{IN}$  and the slope is less than 1

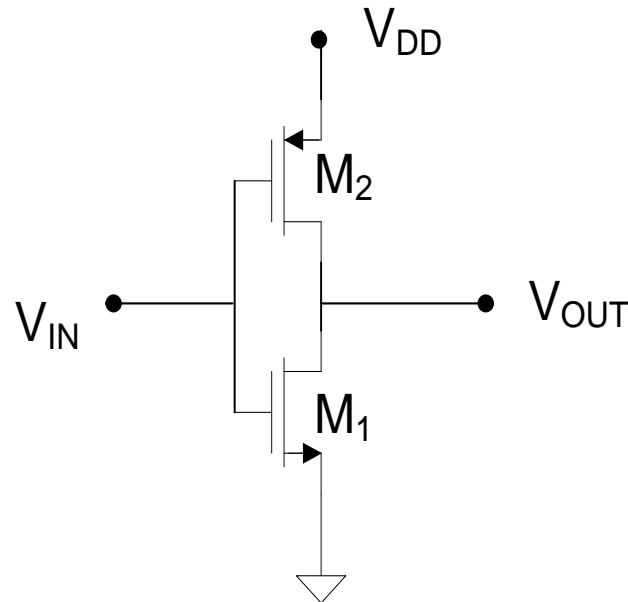
# Inverter Transfer Characteristics of Inverter Pair for THIS Logic Family



$$V_H = V_{DD} \text{ and } V_L = 0$$

Note this is independent of device sizing for THIS logic family !!

# Sizing of the Basic CMOS Inverter



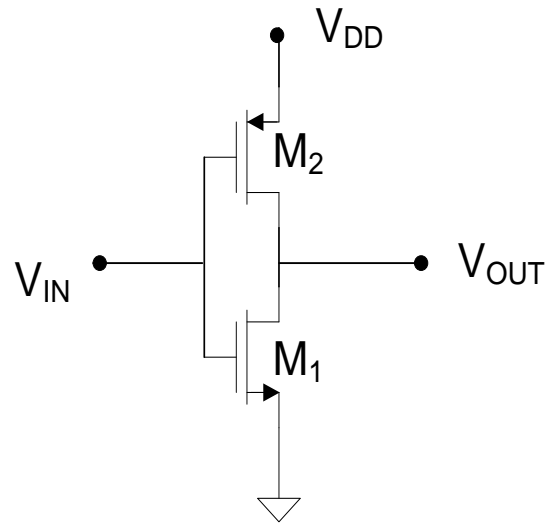
The characteristic that device sizes do not need to be used to establish  $V_H$  and  $V_L$  logic levels is a major advantage of this type of logic

How should  $M_1$  and  $M_2$  be sized?

How many degrees of freedom are there in the design of the inverter?



# How should $M_1$ and $M_2$ be sized?



How many degrees of freedom are there in the design of the inverter?

$$\{ W_1, W_2, L_1, L_2 \}$$

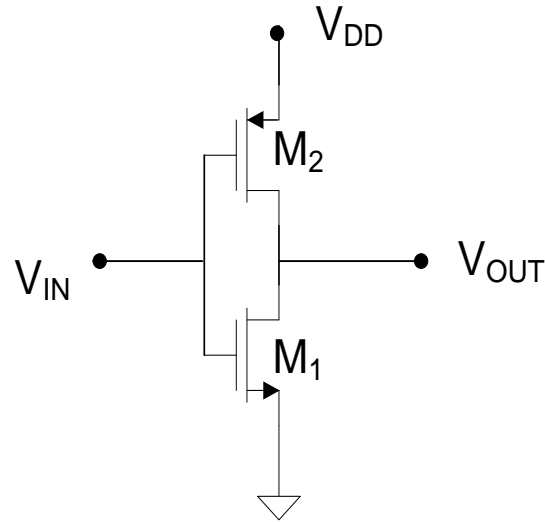
4 degrees of freedom

But in basic device model and in most performance metrics,  $W_1/L_1$  and  $W_2/L_2$  appear as ratios

$$\{ W_1/L_1, W_2/L_2 \}$$

effectively 2 degrees of freedom

# How should $M_1$ and $M_2$ be sized?



$\{ W_1, W_2, L_1, L_2 \}$

4 degrees of freedom

Usually pick  $L_1=L_2=L_{\min}$

That leaves

$\{ W_1, W_2 \}$

effectively 2 degrees of freedom

How are  $W_1$  and  $W_2$  chosen?

Depends upon what performance parameters are most important for a given application!

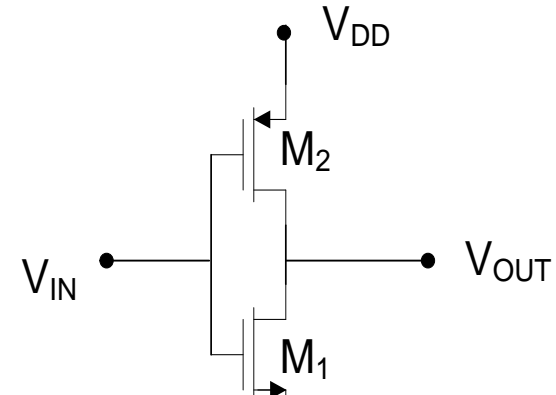
# How should $M_1$ and $M_2$ be sized?

Pick  $L_1=L_2=L_{\min}$

One popular sizing strategy:

1. Pick  $W_1=W_{\min}$  to minimize area of  $M_1$
2. Pick  $W_2$  to set trip-point at  $V_{DD}/2$

Observe Case 3 provides expression for  $V_{\text{TRIP}}$

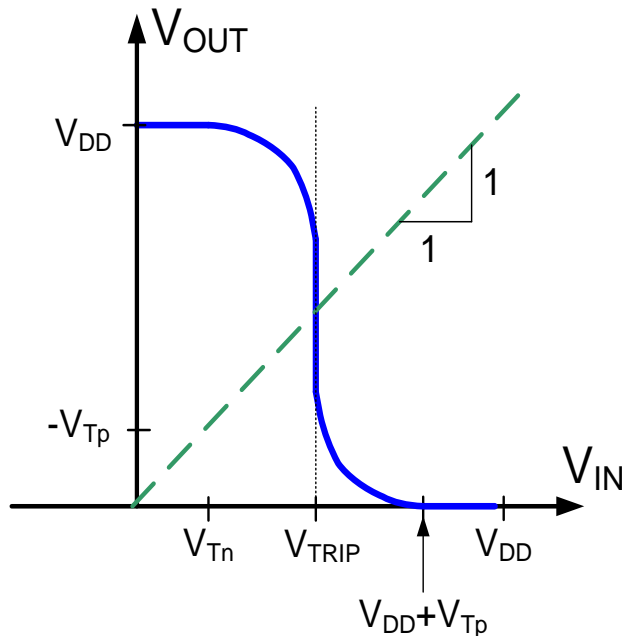


Thus, at the trip point,

$$V_{\text{OUT}} = V_{\text{IN}} = V_{\text{TRIP}} = \frac{(V_{\text{Tn}}) + (V_{\text{DD}} + V_{\text{Tp}}) \sqrt{\frac{\mu_p W_2}{\mu_n W_1}}}{1 + \sqrt{\frac{\mu_p W_2}{\mu_n W_1}}}$$

If  $V_{\text{Tn}} = -V_{\text{Tp}}$

$$\frac{V_{\text{DD}}}{2} = \frac{(V_{\text{Tn}}) + (V_{\text{DD}} - V_{\text{Tn}}) \sqrt{\frac{\mu_p W_2}{\mu_n W_1}}}{1 + \sqrt{\frac{\mu_p W_2}{\mu_n W_1}}}$$



# How should $M_1$ and $M_2$ be sized?

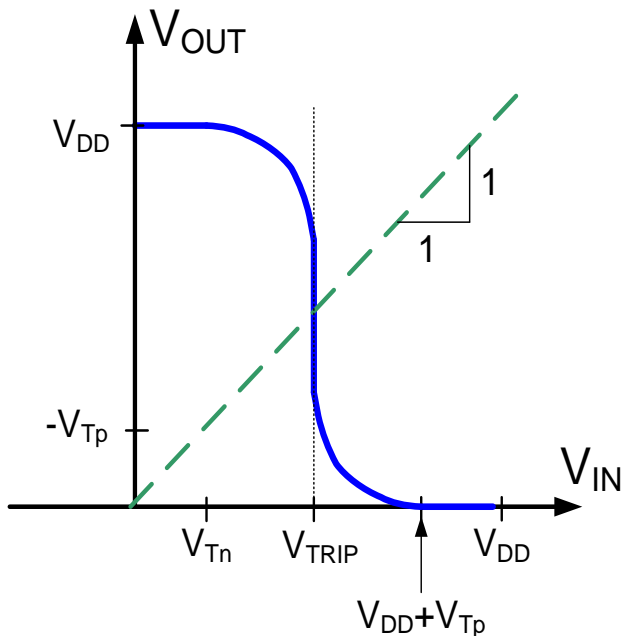
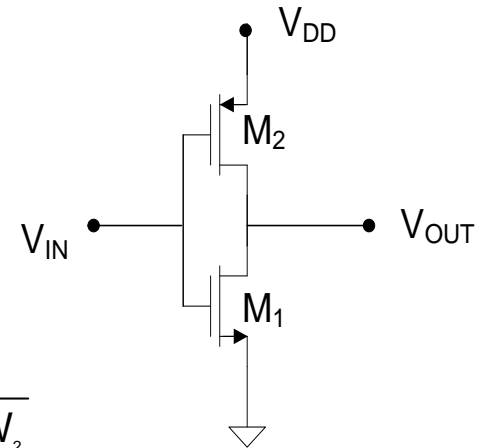
Pick  $L_1=L_2=L_{\min}$

One popular sizing strategy:

1. Pick  $W_1=W_{\min}$  to minimize area of  $M_1$
2. Pick  $W_2$  to set trip-point at  $V_{DD}/2$

Observe Case 3 provides expression for  $V_{\text{TRIP}}$

(solution continued)



$$\frac{V_{DD}}{2} = \frac{(V_{Tn}) + (V_{DD} - V_{Tn}) \sqrt{\frac{\mu_p W_2}{\mu_n W_1}}}{1 + \sqrt{\frac{\mu_p W_2}{\mu_n W_1}}}$$

solving for  $\sqrt{\frac{\mu_p W_2}{\mu_n W_1}}$  we obtain

$$\sqrt{\frac{\mu_p W_2}{\mu_n W_1}} = \frac{V_{Tn} - \frac{V_{DD}}{2}}{-\frac{V_{DD}}{2} + V_{Tn}} = 1$$

thus

$$\frac{W_2}{W_1} = \frac{\mu_n}{\mu_p} \Rightarrow W_2 = \frac{\mu_n}{\mu_p} W_{\min} \approx 3W_{\min}$$

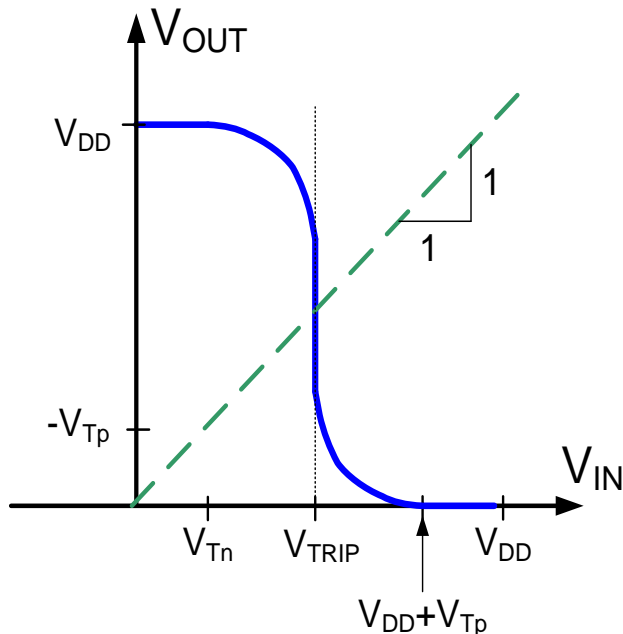
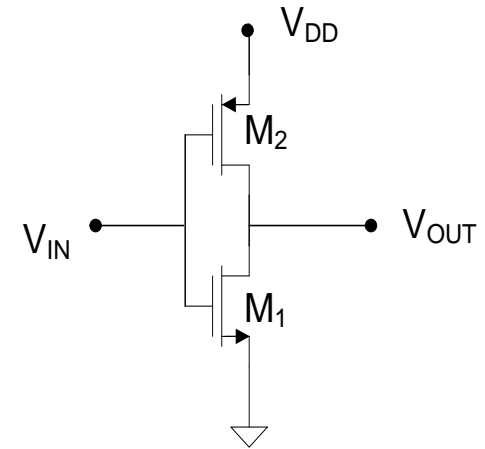
# How should $M_1$ and $M_2$ be sized?

Pick  $L_1=L_2=L_{\min}$

One popular sizing strategy:

1. Pick  $W_1=W_{\min}$  to minimize area of  $M_1$
2. Pick  $W_2$  to set trip-point at  $V_{DD}/2$

Observe Case 3 provides expression for  $V_{TRIP}$



Summary:  $V_{TRIP} = \frac{V_{DD}}{2}$  sizing strategy

$$L_1=L_2=L_{\min}$$

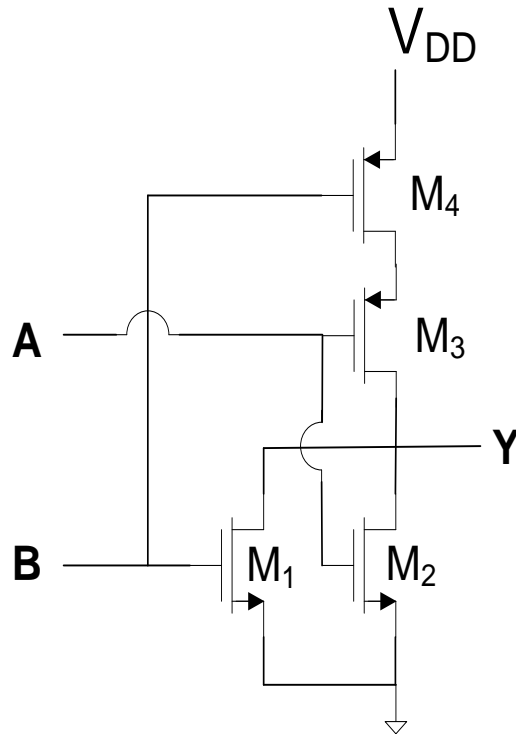
$$W_1=W_{\min}$$

$$W_2 = \frac{\mu_n}{\mu_p} W_{\min} \approx 3W_{\min}$$

(dependent upon assumption  $V_{Tp} = -V_{Tn}$ )

Other sizing strategies will be discussed later !

# Extension of Basic CMOS Inverter to Multiple-Input Gates



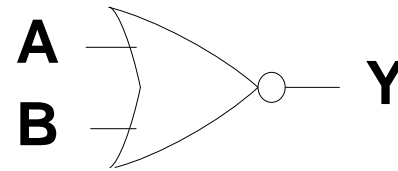
A	B	Y
0	0	1
0	1	0
1	0	0
1	1	0

*Truth Table*

Performs as a 2-input NOR Gate

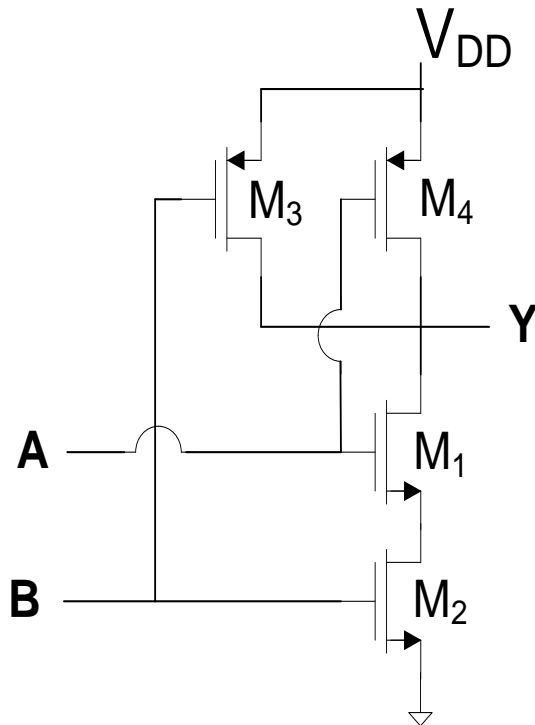
Can be easily extended to an n-input NOR Gate

$V_H = V_{DD}$  and  $V_L = 0$  (inherited from inverter analysis)



analysis not shown here but straightforward and consistent with claim that performance of gates in logic family determined by those of basic inverter

# Extension of Basic CMOS Inverter to Multiple-Input Gates



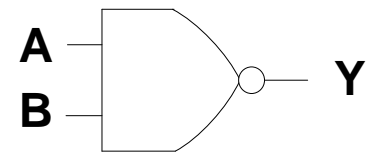
A	B	Y
0	0	1
0	1	1
1	0	1
1	1	0

*Truth Table*

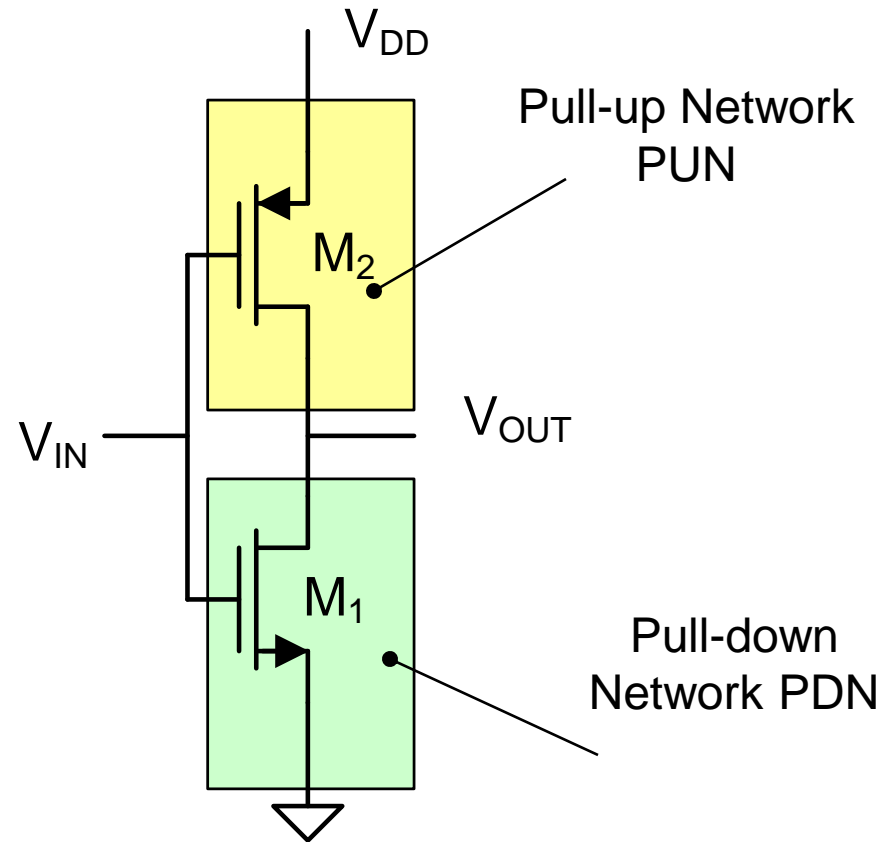
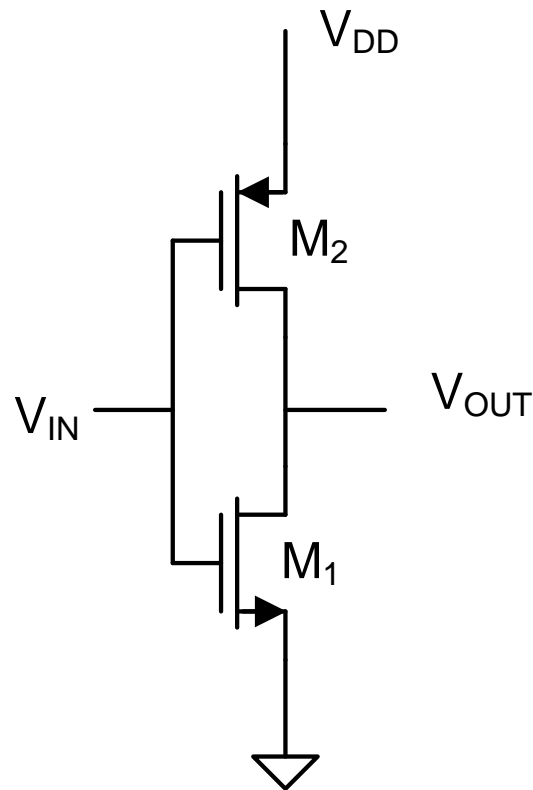
Performs as a 2-input NAND Gate

Can be easily extended to an n-input NAND Gate

$V_H = V_{DD}$  and  $V_L = 0$  (inherited from inverter analysis)



# Static CMOS Logic Family

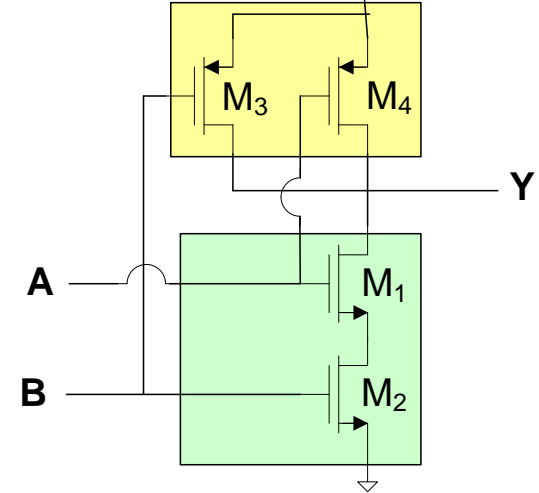
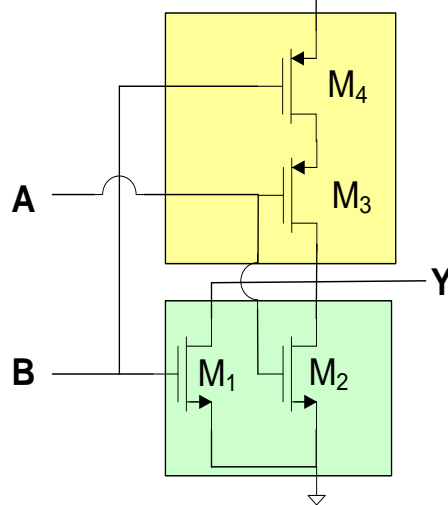
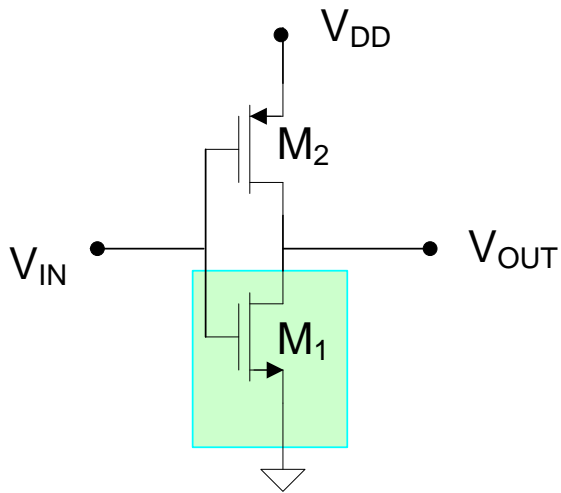
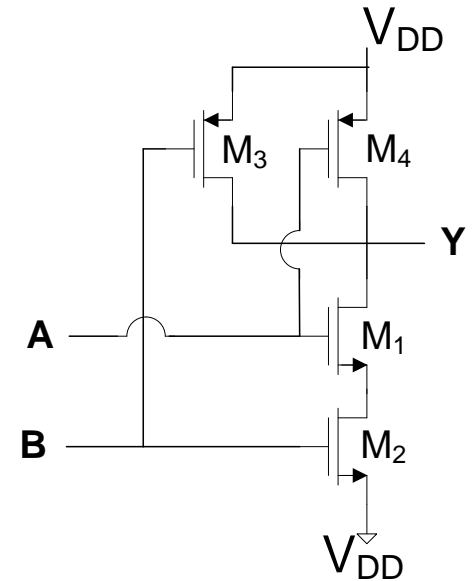
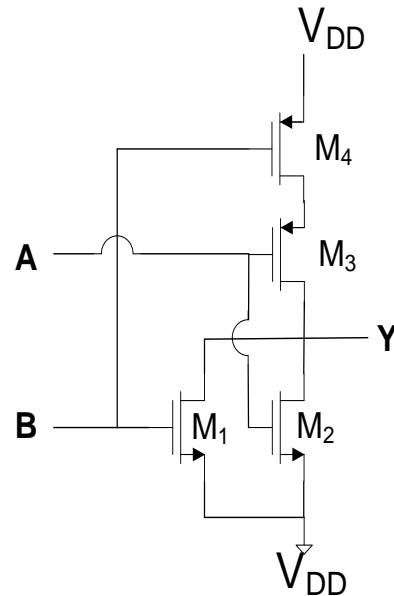
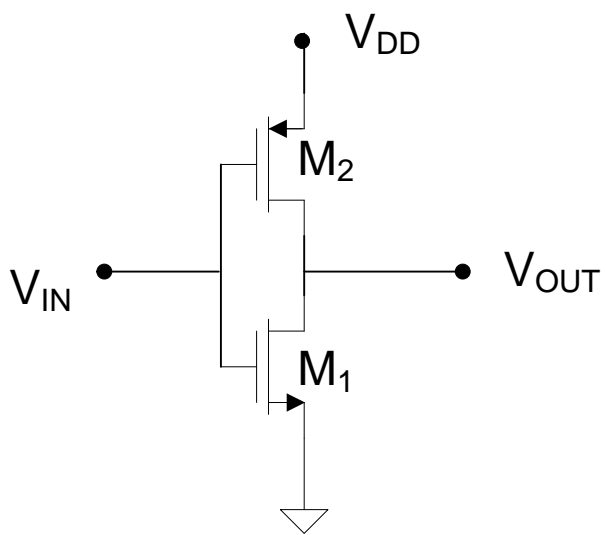


Observe PUN is p-channel, PDN is n-channel

$V_H = V_{DD}$  and  $V_L = 0$  (inherited from inverter analysis)



# Static CMOS Logic Family



n-channel PDN and p-channel PUN

$V_H = V_{DD}$ ,  $V_L = 0V$  (same as for inverter!)



Stay Safe and Stay Healthy !

**End of Lecture 37**